Beyond Asymmetric Choice:
A note on some extensions

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1 Introduction

Free choice Petri nets are a twenty-five year old branch of net theory. Free choice nets are a generalization of state machines (S-systems) and marked graphs (T-systems) for which strong theoretical results hold and efficient analysis techniques exist. However, in many application domains realistic models are not free choice. Therefore, much effort has been devoted to investigating possible generalizations of free choice nets. Examples of net classes for which some of the results for free choice nets have been generalized are: equal-conflict nets [TS93], and well-handled nets [ES90]. Weaker results (typically only one direction of a theorem for free choice nets) have been generalized to larger net classes. A typical example is the work on *asymmetric choice nets* [Hac72, BS87, DE95]. For example one direction of Commoner's Theorem can be generalized to asymmetric choice nets.

In this paper, we explore further generalizations of asymmetric choice nets. In an asymmetric choice net, it is not allowed that two transitions compete for a token in a shared input place while they both have private input places, i.e., the input set of the first transition must be included in the input set of the other transition or vise versa. Clearly, free choice nets satisfy this structural property. In this paper, we extend the notion of asymmetric choice with *test arcs*, also called loops or self loops. If a place is both an input and an output place of a transition, the set of two arcs connecting the place and the transition is called a test arc because the transition only tests the presence of the token and does not really remove it while firing. We will investigate several alternative definitions. It turns out that allowing arbitrary test arcs in addition to the classical definition of asymmetric choice is not suitable. The most basic results do not hold for these naive extensions. Therefore, we give a more sophisticated definition of a net class we call *extended asymmetric choice nets*. For this class, we will show that it is possible to generalize one direction of Commoner's Theorem: *If every proper siphon of an extended asymmetric choice system includes an initially marked trap, the system is live.*

This work is motivated by the fact that in many application domains test arcs are needed to model the behavior of systems and business processes. Test arcs are for example used to test signals in models of transport and production systems. In the workflow management domain, test arcs are often used to model that the execution of one task in one sub-procedure has to wait until another sub-procedure has advanced until a predefined point (milestone). See for example the workflow process definition shown in Figure 15 in [Aal98].

The remainder of this paper is organized as follows. First, we introduce some basic notations and standard results. Then, we give five definitions of (naively) (extended) asymmetric choice nets and show the equivalence of some of these net classes. Finally, we prove some results for one of the net classes identified and discuss the relevance of these results.

2 Basic concepts

In this section, we introduce the notation used in this paper and some of the standard results for Petri nets [Pet81, Rei85, Mur89, DE95]. A Petri net consists of a finite set of *places* P, a finite set of *transitions* T, and a *flow relation* F which relates transitions and places. A *marking* of a net associates a natural number with each place. This number represents the number of *tokens* residing at that place. In a textual representation, we use multi-set notation for markings, e.g. $[A, C, A]$ represents a marking with two tokens at place A, a single token at place C, and no tokens on all other places.

Definition 1 (Net, marking, system net)
Let P and T be two finite and disjoint sets and let F be a relation $F \subseteq (P \times T) \cup (T \times P)$. Then, we call $N = (P, T, F)$ a *net*. A mapping $M : P \longrightarrow \mathbb{N}$ is called a *marking* of N. A net N equipped with a marking M is called a *system net* $\Sigma = (N, M)$ and M is called the *initial marking* of Σ.

The places and transitions of a net are also called the *elements* of the net. For a given element x, the preset $^\bullet x$ denotes all elements which have an arc towards x; the postset $x^\bullet$ denotes all those elements which have an arc coming from x.

Definition 2

Let $N = (P, T, F)$ be a net.

1. For an element $x \in P \cup T$, we define the *preset of* x by $^\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$. We define the *postset of* x by $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$.
2. A set $S \subseteq P$ is called a *siphon of* N, if for every transition $t \in T$ with $t^\bullet \cap S \neq \emptyset$, we also have $^\bullet t \cap S \neq \emptyset$.
3. A set $S \subseteq P$ is called a *trap of* N, if for every transition $t \in T$ with $^\bullet t \cap S \neq \emptyset$, we also have $t^\bullet \cap S \neq \emptyset$.
4. A set of places S is *unmarked at a marking* M, if for every $p \in S$ we have $M(p) = 0$.

The definitions of siphons and traps are structural. Still, there are behavioral consequences (cf. Prop. 5).

A transition is *enabled* at a marking, if every place in its preset is marked. An enabled transition may *occur* in which case one token is removed from every place in the transition's preset and one token is added to every place in the transition's postset.

Definition 3 (Behaviour of nets)
Let $N = (P, T, F)$ be a net and let M be a marking of N.

1. A transition $t \in T$ *is enabled at* M, if for every $p \in {}^\bullet t$ we have $M(p) \geq 1$.
2. If transition $t \in T$ is enabled at M, it may *occur* and its occurrence changes the marking into the *successor marking* M' defined by

$$M'(p) = \begin{cases} M(p) & \text{if } p \notin {}^\bullet t \text{ and } p \notin t^\bullet \\ M(p) & \text{if } p \in {}^\bullet t \text{ and } p \in t^\bullet. \\ M(p) - 1 & \text{if } p \in {}^\bullet t \text{ and } p \notin t^\bullet \\ M(p) + 1 & \text{if } p \notin {}^\bullet t \text{ and } p \in t^\bullet \end{cases}$$

   Then, we write $M \xrightarrow{t} M'$
3. A marking M' is *reachable from* M, if there exists a (possibly empty) sequence of markings $M_1, M_2, \dots, M_{n-1}$ and transitions $t_1, t_2, \dots, t_n$ such that $M \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \rightarrow \dots \xrightarrow{t_n} M'$.

If $M \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \rightarrow \dots \xrightarrow{t_n} M'$, then $\sigma = t_1, t_2, \dots, t_n$ is the occurrence sequence leading from M to M' (notation $M \xrightarrow{\sigma} M'$). With these basic concepts, we are able to define liveness and place liveness.

Definition 4 (Liveness, place liveness)
Let $\Sigma = (N, M)$ be a system net.

1. A transition t of Σ is *dead*, if there is no reachable marking which enables t.
2. A transition t of Σ is *live*, if for every marking M_1 which is reachable from M there exists a marking M_2 which enables t and is reachable from M_1.
3. Σ is *live*, if every transition of Σ is live.
4. A place p of Σ is *dead*, if there is no reachable marking which marks p.
5. A place p of Σ is *live*, if for every marking M_1 which is reachable from M there exists a marking M_2 which marks p and is reachable from M_1.
6. Σ is *place live*, if every place of Σ is place live.

If a transition/place is live, then it is not dead. However, there may be transitions/places which are neither dead nor live. We say that a transition/place of net N is *dead at a marking* M, if and only if, the transition/place is dead in the system $\Sigma = (N, M)$. The following results are well-known in Petri net theory (e.g. [DE95]) and will be used in the second part of this paper.

Proposition 5

Let $\Sigma = (N, M)$ be a system net.

1. Let S be a siphon of N and M_1 be a marking in which S is unmarked. Then, S is unmarked at every marking M_2 reachable from M_1.
2. Let S be a trap of N and M_1 be a marking in which S is marked. Then, S is marked at every marking M_2 reachable from M_1.
3. Let S be a siphon of N and M_1 be a marking which is reachable from M and at which S is unmarked. If there exists a transition t with ${}^\bullet t \cap S \neq \emptyset$, then Σ is not live.

3 Extended asymmetric choice nets

Asymmetric choice nets are a generalization of free choice nets. Therefore, we start with the definition of free choice nets [DE95]. In a free-choice net, a marking that enables a transition t will enable all other transitions sharing an input place with t. The free choice property is a structural property which can be defined in many ways.

Definition 6 A net $N = (P, T, F)$ is *free choice* is for every two places p_1 and p_2 either $p_1{}^\bullet \cap p_2{}^\bullet = \emptyset$ or $p_1{}^\bullet = p_2{}^\bullet$.

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In the following definition we give five possible generalizations of free choice nets. The first two correspond to the notion of asymmetric choice, the other extend this notion with test arcs.

Definition 7 (PAC,TAC,EAC,NEPAC,NETAC) Let $N = (P, T, F)$ be a net.

1. Net N is a *place asymmetric choice (PAC) net*, if for each two places $p_1, p_2 \in P$ with $p_1{}^\bullet \cap p_2{}^\bullet \neq \emptyset$, we have $p_1{}^\bullet \subseteq p_2{}^\bullet$ or $p_2{}^\bullet \subseteq p_1{}^\bullet$.
2. Net N is a *transition asymmetric choice (TAC) net*, if for each triplet of transitions $t, t_1, t_2 \in T$, we have ${}^\bullet t \cap {}^\bullet t_1 \subseteq {}^\bullet t_2$ or ${}^\bullet t \cap {}^\bullet t_2 \subseteq {}^\bullet t_1$.
3. For each transition $t \in T$ we define a relation $\rightsquigarrow_t$ on the places ${}^\bullet t$ as follows. For $p_1, p_2 \in {}^\bullet t$ we have $p_1 \rightsquigarrow_t p_2$, if and only if there exists a transition $t' \in T$ such that $p_1 \in {}^\bullet t' \setminus t'^\bullet$ and $p_2 \notin {}^\bullet t'$.

   Net N is called an *extended asymmetric choice (EAC) net*, if for every transition $t \in T$ the relation $\rightsquigarrow_t$ is acyclic.
4. Net N is a *naively extended place asymmetric choice (NEPAC) net*, if for each two places $p_1, p_2 \in P$ with $p_1{}^\bullet \cap p_2{}^\bullet \neq \emptyset$, we have $(p_1{}^\bullet \setminus {}^\bullet p_1) \subseteq p_2{}^\bullet$ or $(p_2{}^\bullet \setminus {}^\bullet p_2) \subseteq p_1{}^\bullet$.

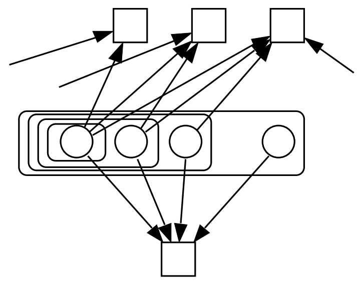

Figure 1: Asymmetric choice nets.

5. Net N is a *naively extended transition asymmetric choice (NETAC) net*, if for each triplet of transitions $t, t_1, t_2 \in T$, we have $({}^\bullet t \cap {}^\bullet t_1) \subseteq (t_1{}^\bullet \cup {}^\bullet t_2)$ or $({}^\bullet t \cap {}^\bullet t_2) \subseteq (t_2{}^\bullet \cup {}^\bullet t_1)$.

In Proposition 8, we will prove that the first two notions (PAC and TAC) and the last two notions (NEPAC and NETAC) are equivalent. PAC and TAC correspond to the usual notion of asymmetric choice [Bes87, DE95]. Some author use the term *extended simple* to denote this class of nets [Hac72, BS87, Bes84]. Figure 1 illustrates the notion of asymmetric choice: The conflict sets of transitions form an ascending chain. NEPAC is a generalization of PAC where test arcs are partially omitted. Note that $p_1{}^\bullet \subseteq p_2{}^\bullet$ implies that $(p_1{}^\bullet \setminus {}^\bullet p_1) \subseteq p_2{}^\bullet$. Similar remarks hold for NETAC and TAC. The definition of extended asymmetric choice (EAC) nets is more involved. If we compare extended asymmetric choice nets with the diagram shown in Figure 1, there may be test arcs violating the ascending chain of conflict sets as long as relation $\leadsto_t$ is acyclic. Note that definition of extended asymmetric choice is not equivalent to any of the other definitions. It is easy to construct an extended asymmetric choice net which is not asymmetric choice and Figure 2 shows a system net Σ_1 which is naively extended asymmetric choice but not extended asymmetric choice because $\leadsto_t$ has a cycle.

Proposition 8 Let $N = (P, T, F)$ be a net.

1. Net N is place asymmetric choice (PAC), if and only if, N is transition asymmetric choice (TAC).
2. Net N is naively extended place asymmetric choice (NEPAC), if and only if, N is naively extended transition asymmetric choice (NETAC).

Proof:

1. First, we prove the "if" direction. Assume that N is not PAC. There are places p_1 and p_2 such that $p_1{}^\bullet \cap p_2{}^\bullet \neq \emptyset$, $p_1{}^\bullet \not\subseteq p_2{}^\bullet$ and $p_2{}^\bullet \not\subseteq p_1{}^\bullet$. Therefore, there are transitions t, t_1 and t_2 such that $t \in p_1{}^\bullet \cap p_2{}^\bullet$, $t_1 \in p_1{}^\bullet \setminus p_2{}^\bullet$, and $t_2 \in p_2{}^\bullet \setminus p_1{}^\bullet$. Hence, $p_1 \in {}^\bullet t$, $p_2 \in {}^\bullet t$, $p_1 \in {}^\bullet t_1$, $p_2 \notin {}^\bullet t_1$, $p_2 \in {}^\bullet t_2$, and $p_1 \notin {}^\bullet t_2$. Combining these properties shows that $p_1 \in ({}^\bullet t \cap {}^\bullet t_1) \setminus {}^\bullet t_2$ and $p_2 \in ({}^\bullet t \cap {}^\bullet t_2) \setminus {}^\bullet t_1$. Therefore,

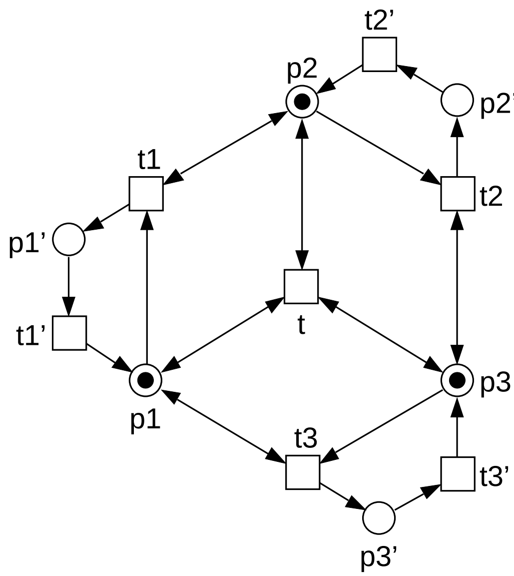

Figure 2: A NEAC system Σ_1 which is not EAC.

N is not TAC since ${}^\bullet t \cap {}^\bullet t_1 \not\subseteq {}^\bullet t_2$ and ${}^\bullet t \cap {}^\bullet t_2 \not\subseteq {}^\bullet t_1$. The "only i" direction can be proved in a similar way. Assume that N is not TAC and prove that N is not PAC.

2. We start by proving the "only if" direction. Assume that N is not NETAC. There are transitions t, t_1 and t_2 such that $({}^\bullet t \cap {}^\bullet t_1) \not\subseteq (t_1{}^\bullet \cup {}^\bullet t_2)$ and $({}^\bullet t \cap {}^\bullet t_2) \not\subseteq (t_2{}^\bullet \cup {}^\bullet t_1)$. Hence, there is a place p_1 such that $p_1 \in {}^\bullet t$, $p_1 \in {}^\bullet t_1$, $p_1 \notin t_1{}^\bullet$, and $p_1 \notin {}^\bullet t_2$. Moreover, there is a place p_2 such that $p_2 \in {}^\bullet t$, $p_2 \in {}^\bullet t_2$, $p_2 \notin t_2{}^\bullet$, and $p_2 \notin {}^\bullet t_1$. Reformulating these terms and combining them shows that $t \in p_1{}^\bullet \cap p_2{}^\bullet$, $t_1 \in (p_1{}^\bullet \setminus {}^\bullet p_1) \setminus p_2{}^\bullet$, and $t_2 \in (p_2{}^\bullet \setminus {}^\bullet p_2) \setminus p_1{}^\bullet$. Hence, N is not NEPAC. The"if" direction can be proved by reversing the order of argumentation.

□

Since PAC and TAC are equivalent, we use the term *asymmetric choice* (AC) in the remainder of this paper. Moreover, we will use the term *naively extended asymmetric choice* (NEAC) instead of NEPAC or NETAC. The proof that PAC and TAC are equivalent was already given in [BS87]. However, Best and Shields give an alternative formulation of TAC (with the addition that $t \in ({}^\bullet t_1)^\bullet \cap ({}^\bullet t_2)^\bullet$).

Proposition 9 Let $N = (P, T, F)$ be a net.

1. If N is asymmetric choice (AC), then N is extended asymmetric choice (EAC).
2. If N is extended asymmetric choice (EAC), then N is naively extended asymmetric choice (NEAC).

Proof:

1. Let $t \in T$ and assume that N is AC. We have to prove that the relation $\rightsquigarrow_t$ defined in Def. 7 is acyclic. Let $p_1, p_2 \in {}^\bullet t$. Since N is AC, $p_1{}^\bullet \subseteq p_2{}^\bullet$ or $p_2{}^\bullet \subseteq p_1{}^\bullet$. Hence, $p_1 \rightsquigarrow_t p_2$ implies that $p_2{}^\bullet \subseteq p_1{}^\bullet$. Clearly, $\rightsquigarrow_t$ is a subset of the relation $R \subseteq P \times P$ with $p_1 R p_2 := p_2{}^\bullet \subseteq p_1{}^\bullet$. Since R is acyclic, $\rightsquigarrow_t$ is also acyclic. Therefore, N is EAC.

2. Assume N is EAC. Let $p_1, p_2 \in S$ be two places with $p_1{}^\bullet \cap p_2{}^\bullet \neq \emptyset$. We will prove that $(p_1{}^\bullet \setminus {}^\bullet p_1) \subseteq p_2{}^\bullet$ or $(p_2{}^\bullet \setminus {}^\bullet p_1) \subseteq p_1{}^\bullet$. There is a transition $t \in p_2{}^\bullet \cap p_2{}^\bullet$. Since N is EAC, it is not possible that $p_1 \rightsquigarrow_t p_2$ and $p_2 \rightsquigarrow_t p_1$ hold ($\rightsquigarrow_t$ is acyclic). Therefore, $p_1 \not\rightsquigarrow_t p_2$ or $p_2 \not\rightsquigarrow_t p_1$. If $p_1 \not\rightsquigarrow_t p_2$, then for all $t' \in T$: $p_1 \notin {}^\bullet t' \setminus t'^\bullet$ or $p_2 \in {}^\bullet t'$. Hence, $(p_1{}^\bullet \setminus {}^\bullet p_1) \subseteq p_2{}^\bullet$. If $p_2 \not\rightsquigarrow_t p_1$, then for all $t' \in T$: $p_2 \notin {}^\bullet t' \setminus t'^\bullet$ or $p_1 \in {}^\bullet t'$. Hence, $(p_2{}^\bullet \setminus {}^\bullet p_2) \subseteq p_1{}^\bullet$. In both cases, we conclude that N is NEAC.

$\square$

If we consider the five classes defined in Definition 7 in conjuncion with Propositions 8 and 9, we see that PAC coincides with TAC, NEPAC coincides with NETAC, PAC and TAC are included in EAC, and EAC is both included in NEPAC and NETAC.

4 Commoner's Theorem for EAC systems

The generalization of one direction of Commoner's Theorem to asymmetric choice systems is due to Commoner, and can be found in Hack's Master Thesis [Hac72, DE95]. In this section, we will show that this result can be generalized to extended asymmetric choice systems but not to naively extended asymmetric choice systems. For this purpose, we prove three propositions that hold for extended asymmetric choice systems.

Proposition 10 Let $\Sigma = (N, M)$ be an extended asymmetric choice system. If transition t is dead at M, then some input place of t is dead at some marking reachable from M.

Proof: We prove the contraposition. Assume that no input place of t is dead at any marking reachable from M. We have to prove that t is not dead at M. Let ${}^\bullet t = \{p_1, \ldots, p_n\}$. The input places of t are ordered in such a way that $i < j$, implies that for all $t' \in T$: if $t' \in p_i{}^\bullet \setminus {}^\bullet p_i$, then $t' \in p_j{}^\bullet$. Note that $p_i \not\rightsquigarrow_t p_j$, if and only if, for all $t' \in T$: if $t' \in p_i{}^\bullet \setminus {}^\bullet p_i$, then $t' \in p_j{}^\bullet$ (see Definition 7). It is possible to order the input places of t in such a way because $\rightsquigarrow_t$ is acyclic. Since no input place of t is dead at any marking reachable from M, there exists an occurrence sequence

$$M \xrightarrow{\sigma_1} M_1 \xrightarrow{\sigma_2} M_2 \rightarrow \ldots \rightarrow M_{n-1} \xrightarrow{\sigma_n} M_n$$

such that $M_i(p_i) > 0$ for $1 \leq i \leq n$. Assume without loss of generality that all sequences σ_i are minimal, i.e., no intermediate marking marks p_i.

We show that M_n marks every place in ${}^\bullet t$, and therefore enables t. We proceed by induction on the index i and prove that, for $1 \leq i \leq n$ and $1 \leq j \leq i$, marking M_i marks place p_j.

$i = 1$: For $i = 1$, $M_i = M_1$, and M_1 marks p_1 by construction.

$i \to i+1$: Let us assume by induction hypothesis that for all $1 \leq j \leq i$, marking M_i marks place p_j. If $j = i+1$, then $p_{i+1} = p_j$ and M_{i+1} marks p_j by construction. If $j < i+1$, then M_i marks p_j (induction hypothesis). By the minimality of $M_i \stackrel{\sigma_{i+1}}{\to} M_{i+1}$, no transition of $p_{i+1}{}^\bullet$ occurs in σ_{i+1}. Since $j < i+1$, we have that for all $t' \in T$: if $t' \in p_j{}^\bullet \setminus {}^\bullet p_j$, then $t' \in p_{i+1}{}^\bullet$. Hence, no transition in $p_j{}^\bullet \setminus {}^\bullet p_j$ occurs in σ_{i+1} (i.e., no tokens are removed from p_j). Therefore, M_{i+1} marks p_j.

Since M_n enables t and M_n is reachable from M, t is not dead at M. □

Note that the proof of Proposition 11 is simular to the proof of Lemma 10.2 in [DE95]. To generalize the proof to extended asymmetric choice systems we have defined another ordering relation on places and modified the induction step.

Proposition 11 An extended asymmetric choice system is live, if it is place live.

Proof: Assume that $\Sigma = (N, M)$ is an extended asymmetric choice system which is not live. There is a reachable marking M' and a transition t such that t is dead at marking M'. By the previous proposition, some input place of t is dead at some marking reachable from M'. Since this marking is also reachable from M, the system is not place live. □

Note that the reverse holds for any system, i.e., live systems are always place live (cf. [DE95]).

Proposition 12
If an extended asymmetric choice system is not place live, there is a proper siphon which is unmarked in some reachable marking.

Proof: Assume that $\Sigma = (N, M)$ is an extended asymmetric choice system which is not place live. Since Σ is not place live, there is a place p and a reachable marking M_1 such that p is dead at M_1. Let M_2 be a marking reachable from M_1 such that every place not dead at M_2 is not dead at any marking reachable from M_2. Such a marking exists, because dead places remain dead, and the set of places in finite. It follows that all markings reachable from M_2 have the same set of dead places, say D. We claim that D is a proper siphon, and that D is unmarked at M_2. We first prove the following three claims:

1. D is not empty.
   The place p is dead at M_1. Since dead places remain dead, p is dead at M_2. So $p \in D$.
2. Every transition t with $t^\bullet \cap D \neq \emptyset$ is dead at M_2.
   Let $q \in D$. Then q is dead at M_2. So every transition in ${}^\bullet q$ is dead at M_2.
3. Every transition t dead at M_2 has an input place in D.
   By Proposition 10, some place $q \in {}^\bullet t$ is dead at a marking reachable from M_2. By the definition of M_2, this place is already dead at M_2, and therefore in D.

Combining these three observations, we conclude that D is a proper siphon. D is unmarked at M_2 because by definition of dead places, every place dead at M_2 is unmarked at M_2. □

Using the three propositions, we can now prove Commoner's Theorem for extended asymmetric choice systems.

Theorem 13 (Commoner's Theorem for EAC)
An extended asymmetric choice system is live, if every proper siphon includes an initially marked trap.

Proof: By contradiction. Assume that the extended asymmetric choice system $\Sigma = (N, M)$ is not live. We will prove that there is a proper siphon which does not contain an initially marked trap. By Proposition 11, we know that Σ is not place live. By Proposition 12, this implies that there is a proper siphon D which is not marked at a state M_2 reachable from M. So every trap included in D is unmarked at M_2. Since traps remain marked, every trap included in D is unmarked in the initial marking M. □

It is quite easy to show that the other direction of Commoner's Theorem does not hold for extended asymmetric choice nets (cf. Figure 10.2 in [DE95]). Moreover, it is not possible to generalize Theorem 13 to naively extended asymmetric choice systems. Consider for example the net shown in Figure 3. The net is naively extended asymmetric choice but not extended asymmetric choice. The net has the following proper siphons: $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and $\{a, b, c\}$. Since $\{a, b\}$, $\{a, c\}$, and $\{b, c\}$ are also traps, every proper siphon includes an initially marked trap. However, it is easy to see that Σ_2 is not live (transition t is dead). This example illustrates that Commoner's Theorem (one direction) cannot be extended to naively extended asymmetric choice systems. This is a consequence of the fact that Proposition 11 does not hold for naively extended asymmetric choice systems, e.g., Σ_2 is place live but not live.

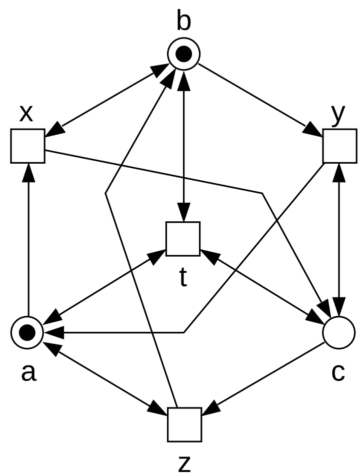

Figure 3: Although every proper siphon includes an intially marked trap, the NEAC system Σ_2 is not live.

5 Conclusion

In this paper, we discussed possible extensions of asymmetric choice nets. The extensions are inspired by the fact that test arcs are an important modeling construct used in many applications domains. Most of the results for asymmetric choice systems have been extended to the class of extended asymmetric choice systems. For example, in Theorem 13 is was shown that one direction of Commoner's Theorem also holds for this class. These results are far from trivial since there is no straightforward translation from extended asymmetric choice nets to asymmetric choice nets. Moreover, we have showed that for more naive extensions of asymmetric choice systems with test arcs, Theorem 13 does not hold.

In another paper [KA98], we show that extended asymmetric choice nets also have some elegant properties with respect to the relation between liveness, fairness and recurrence. If we only consider fair occurrence sequences of an extended asymmetric choice system, then liveness coincides with recurrence (i.e. if transitions can fire, they will fire). This result was already known for free choice nets. The example shown in Figure 2 illustrates that this does not apply to naively extended asymmetric choice nets. Σ_1 is live but not recurrent. Think for example of the following infinite sequence of transition occurrences: $t_1\,(t_2\,t_1'\,t_3\,t_2'\,t_1\,t_3')^\omega$. The corresponding computation is fair but t does not occur.

Both studies (i.e. reported in this paper and [KA98]) show that extended asymmetric choice (EAC) nets, as defined in this paper, are an adequate extension of asymmetric choice nets for which interesting theoretical results hold.

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