

The Imprecisions of Precision Measures in Process Mining

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Abstract

In process mining, *precision measures* are used to *quantify* how much a process model overapproximates the behavior seen in an event log. Although several measures have been proposed throughout the years, no research has been done to validate whether these measures achieve the intended aim of quantifying over-approximation in a consistent way for all models and logs. This paper fills this gap by postulating a number of axioms for quantifying precision consistently for any log and any model. Further, we show through counter-examples that none of the existing measures consistently quantifies precision.

Keywords: Process mining, Formal languages and automata, Petri nets, Design of algorithms

1. Introduction

Process mining [1] is a fast growing discipline that is focused on the analysis of events logged during the execution of a business process. Events contain information on what was done, by whom, for whom, where, when, etc. Such event data are often readily available from information systems such as ERP, CRM, or BPM systems. Process discovery, which plays a prominent role in process mining, is the task of automatically generating a process model that accurately describes a business process based on such event data. Many process discovery techniques have been developed over the last decade (e.g. [2, 3, 4, 5]), producing process models in various forms, such as Petri nets [6], process trees [7], YAWL models [8], and BPMN models [9].

The process model that is pursued by process discovery techniques ideally allows for all the behavior that was observed in the event log (called *fitness*), while at the same time it should not be too general by allowing for much more behavior than what was seen in the event log (called *precision*).

A range of measures have been proposed for quantifying precision [10, 11, 12, 13, 14]. However, to the best of our knowledge, there is currently no work on verifying whether precision measures actually quantify what

they are supposed to measure in a consistent manner. Conceptually, the precision of a process model in the context of an event log should be high when the model allows for few traces not seen in the log, and it should be low when it allows for many traces not seen in the log. In this paper we propose a set of axioms that formulate desired properties of precision measures and systematically validate whether these axioms hold for existing precision measures.

In Section 2 we introduce basic notation and definitions. In Section 3 we formulate axioms for precision measures. We then continue with Section 4, where we describe existing precision measures in more detail and validate the axioms for these measures. In Section 5 we describe two contexts in which we are not able to define axioms for precision. In Section 6 we conclude this paper and state several directions for future work.

2. Preliminaries

In this section we introduce concepts used in later sections of this paper.

$X = \{a_1, a_2, \dots, a_n\}$ denotes a finite set. $\mathcal{P}(X)$ denotes the power set of X , i.e., the set of all possible subsets of X . X^* denotes the set of all sequences over a set X and $\sigma = \langle a_1, a_2, \dots, a_n \rangle$ denotes a sequence of length n , with $\langle \rangle$ the empty sequence. $X \setminus Y$ denotes the set of elements that are in set X but not in set Y , e.g., $\{a, b, c\} \setminus \{a, c\} = \{b\}$. A multiset (or bag) over X is a function $B : X \rightarrow \mathbb{N}$ which we write as $[a_1^{w_1}, a_2^{w_2}, \dots, a_n^{w_n}]$, where for $1 \leq i \leq n$ we have

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$a_i \in X$ and $w_i \in \mathbb{N}^+$. The set of all bags over X is denoted $\mathcal{B}(X)$.

In the context of process mining, we assume the set of all *process activities* Σ to be given. Event logs consist of sequences of events where each event represents a process activity.

Definition 1 (Event, Trace, and Event Log). *An event e in an event log is the occurrence of an activity $e \in \Sigma$. We call a sequence of events $\sigma \in \Sigma^*$ a trace. An event log $L \in \mathcal{B}(\Sigma^*)$ is a finite multiset of traces.*

$L = [\langle a, b, c \rangle^2, \langle b, a, c \rangle^3]$ is an example event log over process activities $\Sigma = \{a, b, c\}$, consisting of 2 occurrences of trace $\langle a, b, c \rangle$ and three occurrences of trace $\langle b, a, c \rangle$.

Most precision measures have been implemented for Petri nets, a process modeling formalism frequently used in the context of process mining. A Petri net is a directed bipartite graph consisting of places (depicted as circles) and transitions (depicted as rectangles), connected by arcs. A transition describes an activity, while places represent the enabling conditions of transitions. Labels of transitions indicate the type of activity that they represent. Unlabeled transitions (τ -transitions) represent invisible transitions (depicted as gray rectangles), which are only used for routing purposes and are not recorded in the event log.

Definition 2 (Labeled Petri net). *A labeled Petri net $N = \langle P, T, F, \ell \rangle$ is a tuple where P is a finite set of places, T is a finite set of transitions such that $P \cap T = \emptyset$, $F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs, called the flow relation, and $\ell: T \rightarrow \Sigma$ is a partial labeling function that assigns a label to a transition, or leaves it unlabeled (the τ -transitions).*

We write $\bullet n$ and $n \bullet$ for the input and output nodes of $n \in P \cup T$ (according to F). A state of a Petri net is defined by its *marking* $m \in \mathcal{B}(P)$ being a multiset of places. A marking is graphically denoted by putting $m(p)$ tokens on each place $p \in P$. State changes occur through transition firings. A transition t is enabled (can fire) in a given marking m if each input place $p \in \bullet t$ contains at least one token. Once t fires, one token is removed from each input place $p \in \bullet t$ and one token is added to each output place $p' \in t \bullet$, leading to a new marking $m' = m - \bullet t + t \bullet$.

A firing of a transition t leading from marking m to marking m' is denoted as step $m \xrightarrow{t} m'$. Steps are lifted to sequences of firing enabled transitions, written $m \xrightarrow{\gamma} m'$ and $\gamma \in T^*$ is a *firing sequence*.

A partial function $f \in X \rightarrow Y$ with domain $\text{dom}(f)$ can be lifted to sequences over X using the following recursive definition: (1) $f(\langle \rangle) = \langle \rangle$; (2) for any $\sigma \in X^*$ and $x \in X$:

$$f(\sigma \cdot \langle x \rangle) = \begin{cases} f(\sigma) & \text{if } x \notin \text{dom}(f), \\ f(\sigma) \cdot \langle f(x) \rangle & \text{if } x \in \text{dom}(f). \end{cases}$$

Defining an *initial* and *final* markings allows to define the *language* accepted by a Petri net as a set of finite sequences of activities.

Definition 3 (Accepting Petri Net). *An accepting Petri net is a triplet $APN = (N, m_0, MF)$, where N is a labeled Petri net, $m_0 \in \mathcal{B}(P)$ is its initial marking, and $MF \subseteq \mathcal{B}(P)$ is its set of possible final markings. A sequence $\sigma \in \Sigma^*$ is a trace of an accepting Petri net APN if there exists a firing sequence $m_0 \xrightarrow{\gamma} m_f$ such that $m_f \in MF$, $\gamma \in T^*$ and $\ell(\gamma) = \sigma$.*

The *language* $\mathcal{Q}(APN)$ is the set of all its traces, i.e., $\mathcal{Q}(APN) = \{\ell(\gamma) \mid \gamma \in T^* \wedge \exists m_f \in MF m_0 \xrightarrow{\gamma} m_f\}$, which can be of infinite size when APN contains loops. Even though we define language for accepting Petri nets, in theory $\mathcal{Q}(M)$ can be defined for any process model M with formal semantics. We denote the universe of process models as \mathcal{M} . For each $M \in \mathcal{M}$, $\mathcal{Q}(M)$ is defined.

For an event log L , $\tilde{L} = \{\sigma \in \Sigma^* \mid L(\sigma) > 0\}$ is the *trace set* of L . For example, for log $L = [\langle a, b, c \rangle^2, \langle b, a, c \rangle^3]$, $\tilde{L} = \{\langle a, b, c \rangle, \langle b, a, c \rangle\}$. For an event log L and a model M we say that L is *fitting* on model M if $\tilde{L} \subseteq \mathcal{Q}(M)$. Precision is related to the behavior that is allowed by a model M that was not observed in the event log L , i.e., $\mathcal{Q}(M) \setminus \tilde{L}$.

3. Axioms for Precision Metrics

The properties that are desired for precision measures are not clearly defined in existing work, although they are often discussed informally. Van der Aalst et al. [15], describe the precision dimension as ‘‘Precision: measure determining whether the model prohibits behavior very different from the behavior seen in the event log. A model with low precision is underfitting.’’. Vanden Broucke et al. [13] describe precision as ‘‘precision (or: appropriateness), i.e., the model’s ability to disallow unwanted behavior;’’. Munoz-Gama and Carmona [12] describe it as ‘‘Precision: refers to overly general models, preferring models with minimal behavior to represent as closely as possible to the log.’’. Buijs et al. [16] describe precision as ‘‘... precision quantifies the fraction of the behavior allowed by the model which is not seen in the event log.’’.

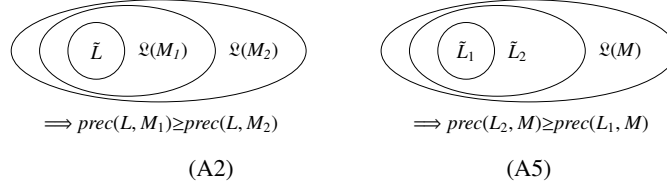


Figure 1: Two of the five axioms for precision measures visualized with Euler diagrams.

We consider precision to be a function $prec(L, M)$ which quantifies which part of the language of model M is seen in event log L . Below we formalize the desired properties of function $prec$ through axioms to consistently hold for any kind of model and any kind of log. Note that in the examples that we will show in this paper all models M will be Petri nets, however the formulated axioms are more general and apply to any process model $M \in \mathcal{M}$. Figure 1 visualizes two axioms using Euler diagrams.

The first axiom states that precision is deterministic, i.e., given a log and model always the same result is returned.

Axiom A1. A precision measure is a function $prec : \mathcal{B}(\Sigma^*) \times \mathcal{M} \rightarrow \mathbb{R}$, i.e., it is deterministic.

Existing precision measures normalize \mathbb{R} to a $[0, 1]$ -interval.

The second axiom formulates the conceptual description of precision more formally: if a process model M_2 allows for more behavior not seen in a log L than another model M_1 does, then M_2 should have a lower precision than M_1 regarding L .

Axiom A2. For models M_1 and M_2 and a log L , $\tilde{L} \subseteq \mathcal{Q}(M_1) \subseteq \mathcal{Q}(M_2) \implies prec(L, M_1) \geq prec(L, M_2)$

Note that **A2** does allow $\tilde{L} \subseteq \mathcal{Q}(M_1) \subset \mathcal{Q}(M_2)$ with $prec(L, M_1) = prec(L, M_2)$. Ideally, since $\mathcal{Q}(M_1)$ is smaller than $\mathcal{Q}(M_2)$ we would like to see a higher precision for M_1 , but this requirement might be too strict. However, for a process model M with $\tilde{L} \subseteq \mathcal{Q}(M)$, we would like the precision of M on L to be higher than the precision of M on any flower model (i.e., a model that allows for all behavior over its activities) on log L .

Axiom A3. For models M_1 and M_2 and a log L , $\mathcal{Q}(M_1) \subset \mathcal{P}(\Sigma^*) \wedge \mathcal{Q}(M_2) = \mathcal{P}(\Sigma^*) \implies prec(L, M_1) > prec(L, M_2)$

The precision of a log on two language equivalent models should be equal, i.e., precision should not depend on the model structure.

Axiom A4. For models M_1 and M_2 and a log L , $\mathcal{Q}(M_1) = \mathcal{Q}(M_2) \implies prec(L, M_1) = prec(L, M_2)$

A4 was stated before in an informal manner by Rozinat and van der Aalst [11], who stated that precision should be independent of structural properties of the model.

Adding fitting traces to a fitting log can only increase the precision of a given model with respect to the log.

Axiom A5. For model M and logs L_1 and L_2 , $\tilde{L}_1 \subseteq \tilde{L}_2 \subseteq \mathcal{Q}(M) \implies prec(L_2, M) \geq prec(L_1, M)$

From **A5** it follows as a corollary that precision is maximal when the log contains all the traces allowed by the model, and minimal when it contains no traces allowed by the model.

In the coming sections we will validate whether these axioms hold for several precision measures. Some articles that introduce precision measures explicitly mention that the measure is intended to be used only with a certain subclass of Petri nets. An example of such a subclass of Petri nets are bounded Petri nets, which have the restriction that all places must have a finite number of tokens in all reachable markings. When an article that introduces a precision measure states an explicit assumption on the subclass of Petri nets, then we only validate the axioms on this subclass of Petri nets. When no explicit assumption on a subclass of Petri nets is stated, we assume that the precision measure is intended for Petri nets in general.

4. Precision Metrics

In this section we give an overview of the precision measures that have been developed in the process mining field, and validate the axioms for precision measures introduced in Section 3 for each of those measures.

4.1. Soundness

Greco et al. [10] were the first to propose a precision measure, defining it as the number of unique executions of the process that were seen in the event log divided by the number of unique paths through the process model. This measure is not usable in practice, because it is zero when the process model allows for an infinite number of paths through the model. Any process model having a loop has a precision of 0. More recent precision

measures are capable of calculating the precision of a model for an event log even when the models allows for infinite behavior.

4.2. Behavioral Appropriateness

Rozinat and Van der Aalst [11] proposed the *simple behavioral appropriateness* precision measure, which looks at the average number of enabled transitions during replay. The authors observed themselves that simple behavioral appropriateness is dependent on the structure of the model, and not solely dependent on the behavior that it allows, therefore **A4** does not hold for this measure. Furthermore, for a process model that contains silent transitions or duplicate labels it is possible that a given trace can be replayed on this model in multiple ways, where the average number of enabled transitions can depend on the chosen replay path through the model. This replay path through the model is chosen arbitrarily from the possible ways in which the trace can be replayed. This shows that **A1** does not hold for simple behavioral appropriateness, as it is not deterministic.

In the same paper, Rozinat and van der Aalst [11] propose *advanced behavioral appropriateness*, which is independent of the model structure. Advanced behavioral appropriateness calculates the sets $S_F \subseteq \Sigma \times \Sigma$ of pairs of activities that sometimes, but not always, follow each other. Likewise set $S_P \subseteq \Sigma \times \Sigma$ is calculated as the set of activities that sometimes, but not always, precede each other. S_F^L and S_P^L denote the sometimes-follows and sometimes-precedes relations on the log, and S_F^M and S_P^M denotes the sometimes-follows and sometimes-precedes relations according to the model. However, to calculate S_F^M and S_P^M , exhaustive exploration of the state space of the model is required, prohibiting the application of this measure for large models or highly concurrent models, where the state-space explosion problem arises. Advanced behavioral appropriateness precision is defined as $a'_b = (\frac{|S_F^L \cap S_F^M|}{2 \cdot |S_F^M|} + \frac{|S_P^L \cap S_P^M|}{2 \cdot |S_P^M|})$. Because S_F^M and S_P^M are obtained through exhaustive exploration of the state space of the model, it is easy to see that they depend only on the behavior of the model and not on its structure, therefore **A4** holds. A problem with advanced behavioral appropriateness occurs for deterministic models, where $|S_P^M| = |S_F^M| = 0$, leading to undefined precision. This shows that advanced behavioral appropriateness is a partial function, which is in conflict with **A1**.

Rozinat and van der Aalst [11] state that simple behavioral appropriateness and advanced behavioral appropriateness assume the Petri net to be in the class of *sound workflow (WF) nets* [17]. A WF-net requires the Petri net to have (i) a single *Start* place, (ii) a single

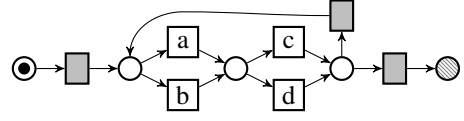


Figure 2: Model M .

End place, and (iii) every node must be on some path from *Start* to *End*. The *soundness* property additionally require that each transition can be potentially executed, and that the process can always terminate properly, i.e., finish with only one token in the *End* place.

Consider model M of Figure 2, which belongs to the class of sound WF-nets, and any log L such that $\tilde{L} \subseteq \mathcal{Q}(M)$. The loop in model M causes S_F^M and S_P^M to contain all pairs of activities of Σ . Therefore, $|S_F^M|$ and $|S_P^M|$ are identical to the sometimes relations $|S_F^{M'}|$ and $|S_P^{M'}|$ of any model M' with $\mathcal{Q}(M') = \mathcal{P}(\Sigma^*)$, leading to $prec(L, M) = prec(L, M')$. As $\mathcal{Q}(M) \subset \mathcal{P}(\Sigma^*)$, this is in conflict with **A3**.

4.3. Escaping Edges Precision

Escaping Edges Precision (ETC) [12] calculates precision by constructing a *prefix automaton*, which consists of one state per unique prefix of the event log. Figure 3b shows an example prefix automaton for an event log $L = [\langle a, c \rangle, \langle a, d \rangle]$. For each state in the prefix automaton it is then determined which activities are allowed as next activities by the process model. Activities that are allowed as next activities for some prefix but that are never observed in the event log after this prefix are referred to as *escaping edges*.

In later work [18, 19], alignments [20] are used to calculate the prefix automaton on the aligned event log instead of the original event log, making the precision measure robust to non-fitting traces, i.e., traces that are not in the language of the model. For a trace σ from a log L that is fitting on an accepting Petri net APN, alignments [20] give a sequence of transition firings $\gamma \in T^*$ such that $m_0 \xrightarrow{\gamma} m_f$ with m_0 the initial marking and m_f a final marking of APN and $\ell(\gamma) = \sigma$. Note that for a given trace σ and model, multiple possible alignments can exist. For non-fitting traces, alignments search for a firing sequence $\gamma \in T^*$ such that $\ell(\gamma)$ is as close as possible to σ . Adriansyah et al. [18] describe two versions of the alignment-based escaping edges precision: *one-align ETC*, which calculates the precision based on one optimal alignment of log and model, and *all-align ETC*, which calculates the precision based on all optimal alignments between log and model. In practice, it is often computationally infeasible to calculate all optimal alignments. A later

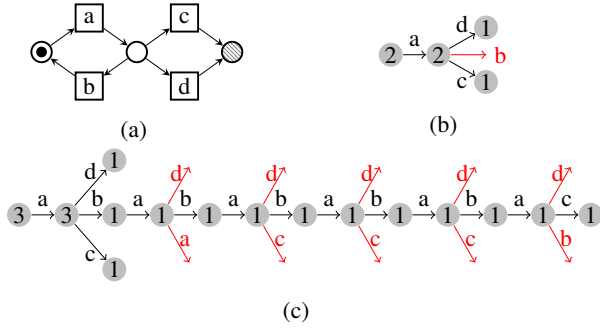


Figure 3: (a) Model M , and the alignment automata on Model M for (b) $\log L_1 = [\langle a, c \rangle, \langle a, d \rangle]$, and for (c) $\log L_2 = [\langle a, c \rangle, \langle a, d \rangle, \langle a, b, a, b, a, b, a, b, a, b, a, c \rangle]$. Red arcs correspond to escaping edges.

precision measure, *representative-align ETC* [21], calculates the escaping edges based on a sample of optimal alignments, and can therefore be seen as a trade-off between the computational efficiency of one-align ETC and the reliability of all-align ETC. The papers on ETC precision and its variants do not state an assumption on a subclass of Petri nets. ETC, one-align ETC, all-align, and representative-align ETC precision are all implemented in the package ETConformance¹ as part of the process mining framework ProM [22].

The one optimal alignment that is used by one-align ETC is chosen arbitrarily from the set of optimal alignments of a log on a model. However, different optimal alignments result in different prefix automata, which can potentially lead to different precision values. This shows that **A1** does not hold for one-align ETC.

Consider $\log L_1 = [\langle a, c \rangle, \langle a, d \rangle]$, $\log L_2 = [\langle a, c \rangle, \langle a, d \rangle, \langle a, c \rangle, \langle a, b, a, b, a, b, a, b, a, b, a, c \rangle]$ and model M be the Petri net of Figure 3a. Note that $\tilde{L}_1 \subset \tilde{L}_2$. The alignment automata generated for the calculation of $prec(L_1, M)$ and the calculation of $prec(L_2, M)$ are shown in Figure 3b and Figure 3c. The circles represent the states of the automaton, and the arrows the transitions. The numbers in the states represent the weights of the states for the precision calculation, i.e., the number of times that states are visited in the alignment of $\log L$ on model M [18]. In an alternative definition of one-align ETC [19] the states are weighted by the number of times that events occurred while being in this state according to the alignment of L on M , instead of the number of times that this state was reached according to this alignment. Figure 3b shows that the initial state was visited twice, activity a occurred twice at the start in $\log L_1$, resulting in a state from which activities b , d , and c were enabled.

¹<https://svn.win.tue.nl/trac/prom/browser/Packages/ETConformance>

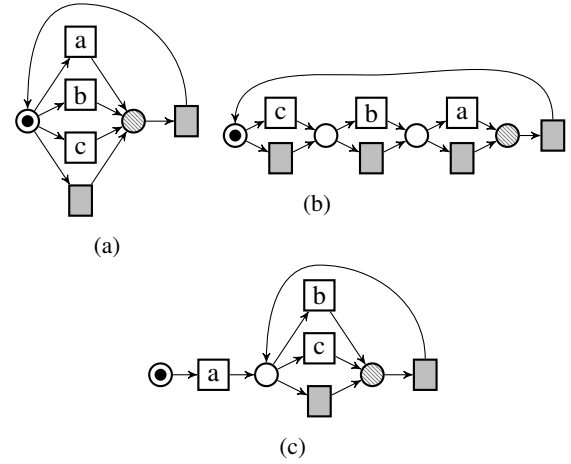


Figure 4: (a) The flower-model, which allows for all behavior over its set of activities, (b) an alternative representation of the flower-model, and (c) a constrained model which always starts with activity a .

From this state, activities c and d were seen once, and activity b was never seen, thus it is an escaping edge. Escaping edges precision is then the (weighted) average ratio of non-escaping edges from all outgoing edges, where states are weighted by the number of times that they are visited. Counting the weighted number of non-escaping edges in the numerator and the weighted total number of edges in the denominator in our example, we find $prec(L_1, M) = \frac{2 \times 1 + 2 \times 2 + 1 \times 0 + 1 \times 0}{2 \times 1 + 2 \times 3 + 1 \times 0 + 1 \times 0} = \frac{6}{8} = 0.75$. One-align ETC results in the following precision values for M on L_1 and L_2 : $prec(L_1, M) = 0.75$ and $prec(L_2, M) = 0.7143$. This shows that **A5** does not hold for one-align ETC. By comparing the automata of Figures 3b and 3c it becomes clear that the single trace that is in L_2 but not in L_1 brings the model to many states with three escaping edges, reducing precision. The prefix automata and the precision calculations for M on logs L_1 and L_2 were performed manually following the procedure from the paper and validation using the ETConformance plugin in ProM.

Now consider $\log L = [\langle a, b, c \rangle]$, and the three Petri nets M_1 , M_2 , M_3 in Figures 4a, 4b, and 4c respectively. Note that M_1 and M_2 are language equivalent, as $\mathcal{L}(M_1) = \mathcal{L}(M_2) = \{a, b, c\}^*$. M_3 is more behaviorally constrained than M_1 and M_2 , since all its traces start with activity a . The one-align precision of M_1 , M_2 , M_3 on L are: $prec(L, M_1) = 0.3333$, $prec(L, M_2) = 0.5238$, and $prec(L, M_3) = 0.4444$. $\mathcal{L}(M_3) \subset \mathcal{L}(M_1)$, but $prec(L, M_3) > prec(L, M_1)$, implying that **A2** does not hold for one-align ETC. Furthermore, $\mathcal{L}(M_1) = \mathcal{L}(M_2)$, but $prec(L, M_1) \neq prec(L, M_2)$, implying that **A4** does not hold for one-align ETC.

Analyzing the ETConformance plugin in ProM we

found that the prefix automaton generated for one-align precision for calculation of $prec(L, M_1)$ results in 6 states, belonging to 3 firings of observable transitions and 2 firings of τ -transitions. In 3 of the 6 states, which correspond to M_1 being in the initial marking, there are 4 possible next activities according to the model, of which only one is observed for that prefix. Furthermore, it shows that the alignment automaton generated for L and M_1 consists of 6 states, the automaton for L and M_2 consists of 12 states, and the automaton for L and M_3 consists of 5 states. This shows that the silent (τ) transitions in M_2 generate additional states in the alignment automaton, leading to a higher precision value.

Computing the precision of M_1 or of M_2 on L did not finish with all-align ETC and representative-align ETC after 8 hours of computation time. The long computation time of all-align ETC and representative-align ETC on models where many optimal alignments exist is a known issue which hinders the application of those measures in practice.

4.4. Negative Event Precision

Goedertier et al. [2] proposed a method to induce *negative events*, i.e., sets of events that were prevented from taking place. Negative events are induced for each position in the event log, i.e., for each event e in each trace of the log a set of events is induced that could not have taken place instead of event e . De Weerd et al. [23] proposed a precision measure based on negative events, *behavioral precision* (p_B), which is closely linked to how precision is defined in the area of data mining. Negative event precision regards a process model as a binary classifier that determines whether a certain event can take place given a certain prefix, and then evaluates the precision of this classifier in data mining terms taking the induced negative events as ground truth. For a given trace prefix, *true positive* (TP) events are defined as events that are possible according to both the process model (i.e., a transition labeled with this event is enabled) and log (i.e., this event is not a negative event). *False positive* events (FP) are negative events induced for a given prefix that were possible according to the model. Behavioral precision is defined as $p_B = \frac{TP}{TP+FP}$, which is in accordance to the definition of precision in the data mining field. In later work [24] induction of artificial negative events has been refined based on frequent temporal patterns which are mined from the event log. Finally, weighted artificial events, where negative events are weighted according to their confidence, are proposed in [13].

Weighted behavioral precision induces negative events for an event e in the log by taking a window

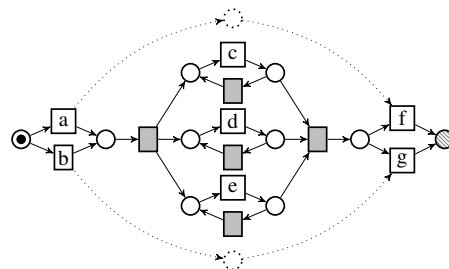


Figure 5: A process model M_1 and its behaviorally restricted variant M_2 which includes the dotted places and arcs.

of events w that directly precede e , then calculating all subsequences of events in the log that exactly match w , and finally negative events are identified by calculating which events have never occurred in the log directly after any subsequence matching w . This procedure is repeated for different windows sizes, and the resulting negative event are weighted by window size.

To induce the events that could not have happened after e.g. trace prefix $\sigma' = \langle a, c, c, d, e, c, d, e, e \rangle$, the method to induce weighted negative events described in [13] searches for subsequences of events in the log that are identical to the latest k events of σ' in the event log. All the activities that have never succeeded such subsequences are considered to be negative events, furthermore, the confidence of these negative events is based on the length k of those matching subsequences.

Negative event based precision measures, with the different methods for negative event induction, are implemented in the ProM package NEConformance². In this paper we evaluate the precision measure that uses weighted negative events [13], which is the most recent approach to induce negative events and the recommended approach for measuring precision [13]. No assumption on a subclass of Petri nets is stated in the papers on negative event precision.

Consider models M_1 and M_2 of Figure 5 respectively excluding and including the arcs and places indicated in dotted lines. $\mathcal{Q}(M_2) \subset \mathcal{Q}(M_1)$, since M_2 contains a long term dependency between activities a and f and between activities b and g , which M_1 does not have. Consider an event log L which consists of 10 traces from M_2 , leading to L being fitting on both M_1 and M_2 . We found the negative event precision of M_1 and M_2 on the same L to be non-deterministic, resulting in slightly different values every time that it is calculated. This shows that **A1** does not hold for negative event precision.

Because negative event precision is non-deterministic, we calculated the precision of M_1

²<http://processmining.be/neconformance/>

and M_2 on L both 20 times. The highest precision found in 20 repetitions for M_1 is 0.4876, while the lowest precision found for M_2 is 0.4545, showing that the non-determinism has the effect that **A2** does not hold for negative event precision. We found an average value of 0.4744 with a standard deviation of 0.0090 for the precision of M_1 on L and an average value of 0.4640 with a standard deviation of 0.0072 for the precision of M_2 on L . This shows that also in terms of average precision value **A2** does not hold.

To test whether the difference in mean precision between M_1 and M_2 is due to chance alone we formulate a null hypothesis:

H_0 : The average negative event precision of M_2 on L is higher than or equal to the average negative event precision of M_1 on L .

Testing this null hypothesis with a one-tailed Welch t -test [25] we found a p -value of 0.0001801, indicating that we can reject the null hypothesis with significance level 0.01. This shows that, with statistical significance, the precision of M_1 on L is higher than the precision of M_2 on L , which is in disagreement with **A2**.

To see why **A2** does not hold for negative event precision, consider the negative event inducing procure being applied to trace prefix $\sigma' = \langle a, c, c, d, e, c, d, e, e \rangle$ from log L . Petri net M_2 generates many different traces because of the parallel length-one-loops on activities c , d and e , which allows for any sequence of any length over these activities. Therefore, the matching subsequences of σ' in the log generated from M_2 are the subsequences that by chance ended in the same behavior over c , d , and e . Because the sequences of c , d and e events can be long and diverse, activity a and b are unlikely to be present in the matching subsequences, which makes it unlikely that the procedure can induce the negative event g for σ' . Because the negative events that reflect the constraint that M_2 introduces compared to M_1 cannot be induced from the log, negative event precision is not able to recognize that M_2 is more precise than M_1 .

4.5. Projected Conformance Checking

Projected Conformance Checking (PCC) precision was developed by Leemans et al. [14] as a computationally efficient precision measure that scales to event logs with billions of events. PCC precision projects both event log and model on all subsets of activities of size k , and generates minimal deterministic finite automata (DFA) for the behavior over these subsets of activities in the log (i.e., *log automaton*) and for the behavior over these events allowed by the model (i.e., *model automaton*). Based on the log automaton and model automaton it then builds a *conjunction automaton* which allows the

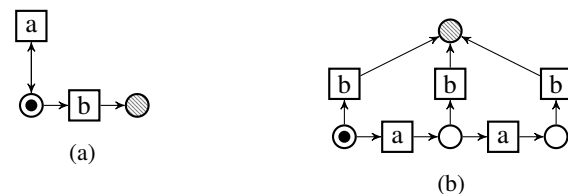


Figure 6: (a) A model with a length-one-loop, (b) the same model with the loop unrolled up to two executions.

behavior that was allowed both in log and model automaton. It then iterates over the states of the model automaton, and calculates over the share of transitions of this state that is also possible in the corresponding state in the conjunction automaton. It defines precision as the average of this share over the model automaton states.

PCC precision is implemented in the ProM package ProjectedRecallAndPrecision³. PCC precision assumes the Petri net to be of the class of bounded Petri nets, i.e., the Petri net for which precision is calculated must have a finite number of tokens in every place for all reachable markings.

Consider log $L = [\langle a, b \rangle]$ and Petri nets M_1 and M_2 of Figures 6a and 6b respectively. M_1 starts with a length-one-loop on activity a , followed by activity b . M_2 unrolls the length-one-loop on activity a of M_1 to at most two executions, thereby limiting the behavior as it only allows at most two executions of activity a . It is easy to see that M_1 and M_2 both belong to the class of bounded Petri nets, as in both models each place can have at most one token. For this log and these models, PCC precision results in $prec(L, M_1)=0.6$, and $prec(L, M_2)=0.5$. However, since $\mathcal{L}(M_2) \subset \mathcal{L}(M_1)$, **A2** states that the precision of M_2 for fitting log L should be higher or equal to its precision of M_1 . This shows that **A2** does not hold for PCC precision.

This drop in precision is an effect of the additional states that are created in the model DFA as an effect of unrolling the length-one-loop. The model DFA created from Petri net M_2 (Figure 6b) for example contains a state s that is reached after firing $\langle a, a \rangle$. This state however is never reached based on event log L , which only contains a trace $\langle a, b \rangle$, which has the effect that none of the enabled transitions from state s were observed in the log, bringing down the precision. In the DFA generated from Petri net M_1 (Figure 6a), this state s is merged with the state that one reaches after observing a single a

³<https://svn.win.tue.nl/trac/prom/browser/Packages/ProjectedRecallAndPrecision/>

discovered that none of the existing precision measures fulfills all formulated requirements.

In future work, we would like fill the empty cells of Table 1 and get a complete overview of the axioms that hold for each precision measure. Furthermore, we would like to use the insights learned from evaluating the axioms on the measures to either repair one of the existing measures or come up with a completely new measure that fulfills all five axioms.

Reproducibility. The event logs and process models that are used as part of a counterexample for a combination of an axiom and a precision measure can be found at [26].

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