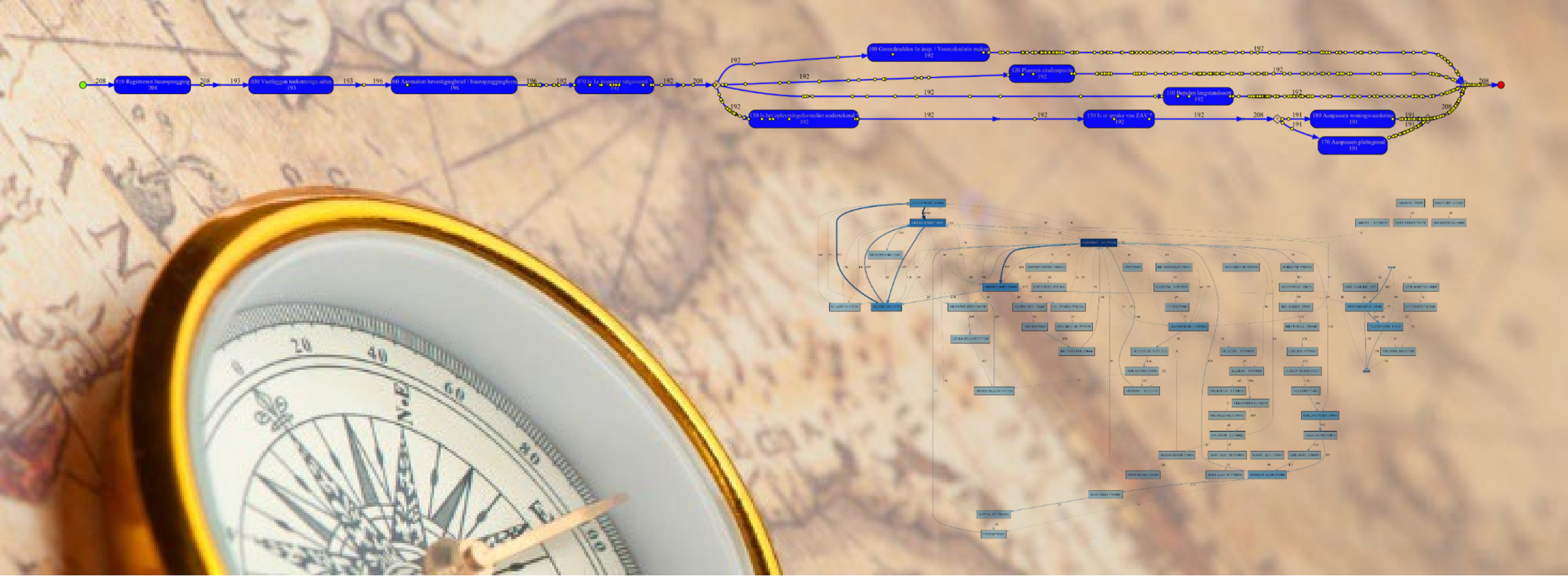

Free-Choice Nets with Home Clusters are (Surprisingly) Lucent

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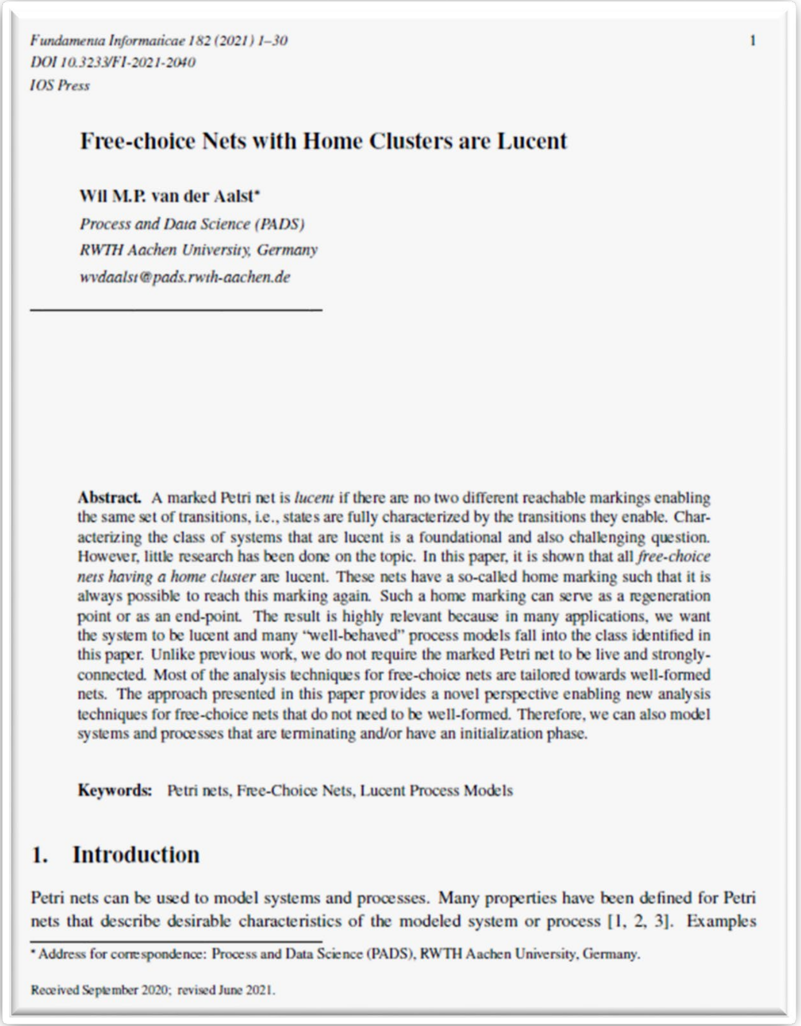

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Free-choice Nets with Home Clusters are Lucent

Wil M.P. van der Aalst*
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Abstract. A marked Petri net is *lucent* if there are no two different reachable markings enabling the same set of transitions, i.e., states are fully characterized by the transitions they enable. Characterizing the class of systems that are lucent is a foundational and also challenging question. However, little research has been done on the topic. In this paper, it is shown that all *free-choice nets having a home cluster* are lucent. These nets have a so-called home marking such that it is always possible to reach this marking again. Such a home marking can serve as a regeneration point or as an end-point. The result is highly relevant because in many applications, we want the system to be lucent and many "well-behaved" process models fall into the class identified in this paper. Unlike previous work, we do not require the marked Petri net to be live and strongly-connected. Most of the analysis techniques for free-choice nets are tailored towards well-formed nets. The approach presented in this paper provides a novel perspective enabling new analysis techniques for free-choice nets that do not need to be well-formed. Therefore, we can also model systems and processes that are terminating and/or have an initialization phase.

Keywords: Petri nets, Free-Choice Nets, Lucent Process Models

1. Introduction

Petri nets can be used to model systems and processes. Many properties have been defined for Petri nets that describe desirable characteristics of the modeled system or process [1, 2, 3]. Examples

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- **Explores the relationship between home clusters and lucency.**
- **For free-choice nets that do not need to be well-formed!**
- **New concepts not building on existing free-choice theory.**

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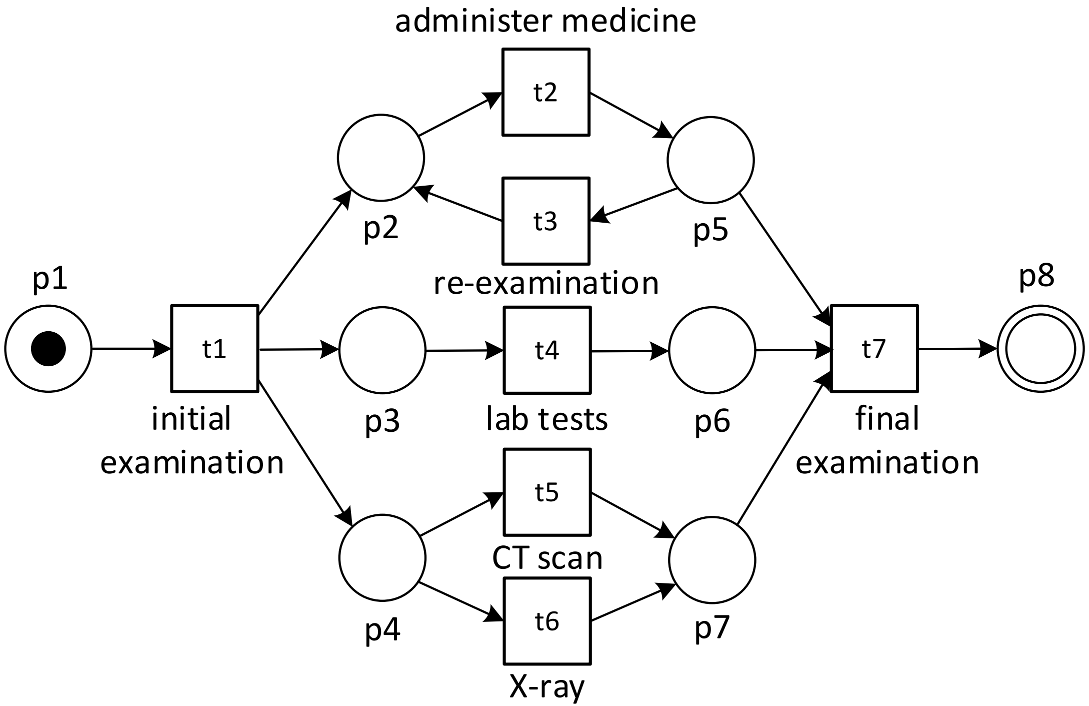
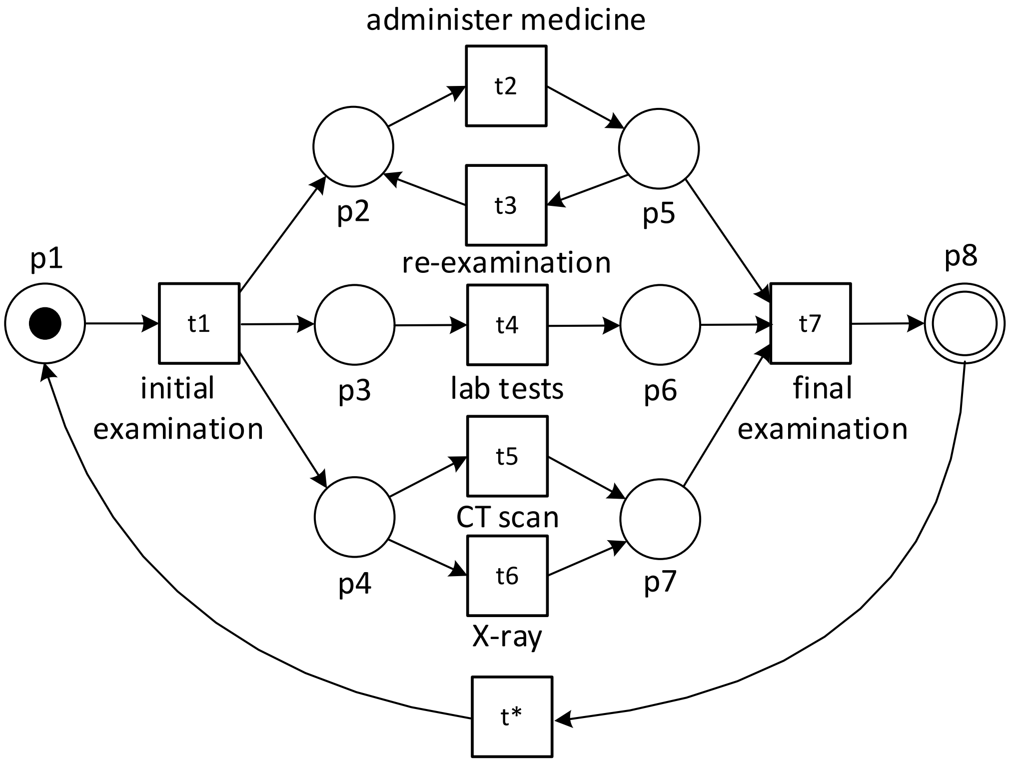
Petri nets: A sound workflow net



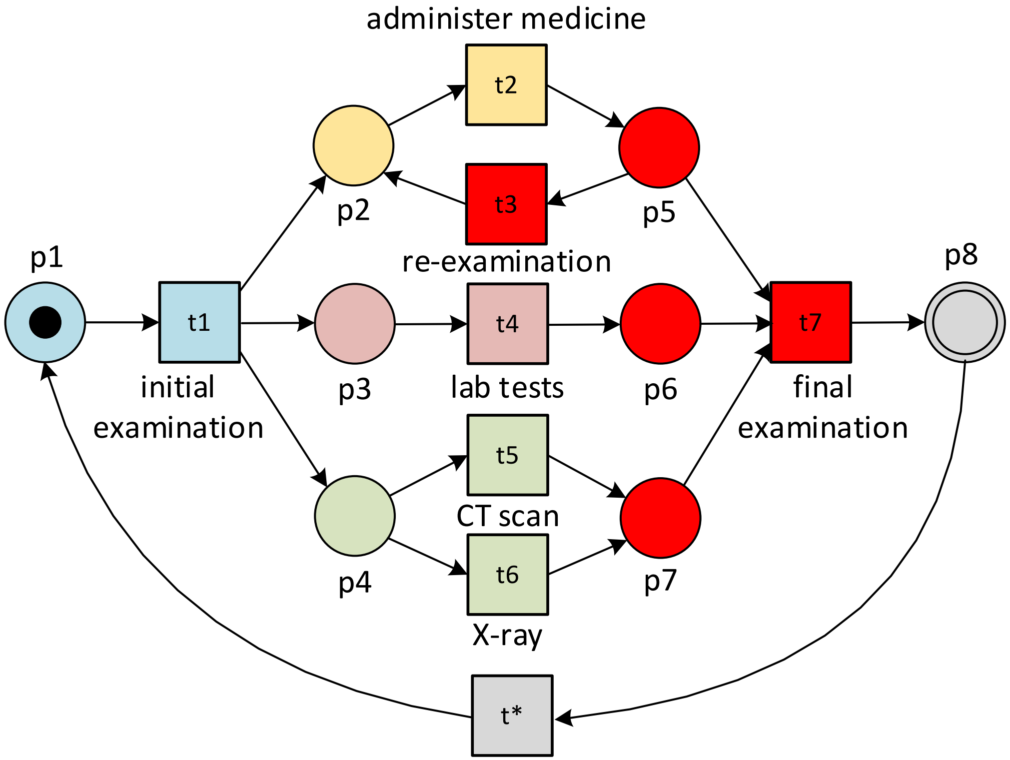
places
transitions
marking
enabling
input/output
firing/occur

Petri nets: A live and safe short-circuited workflow net



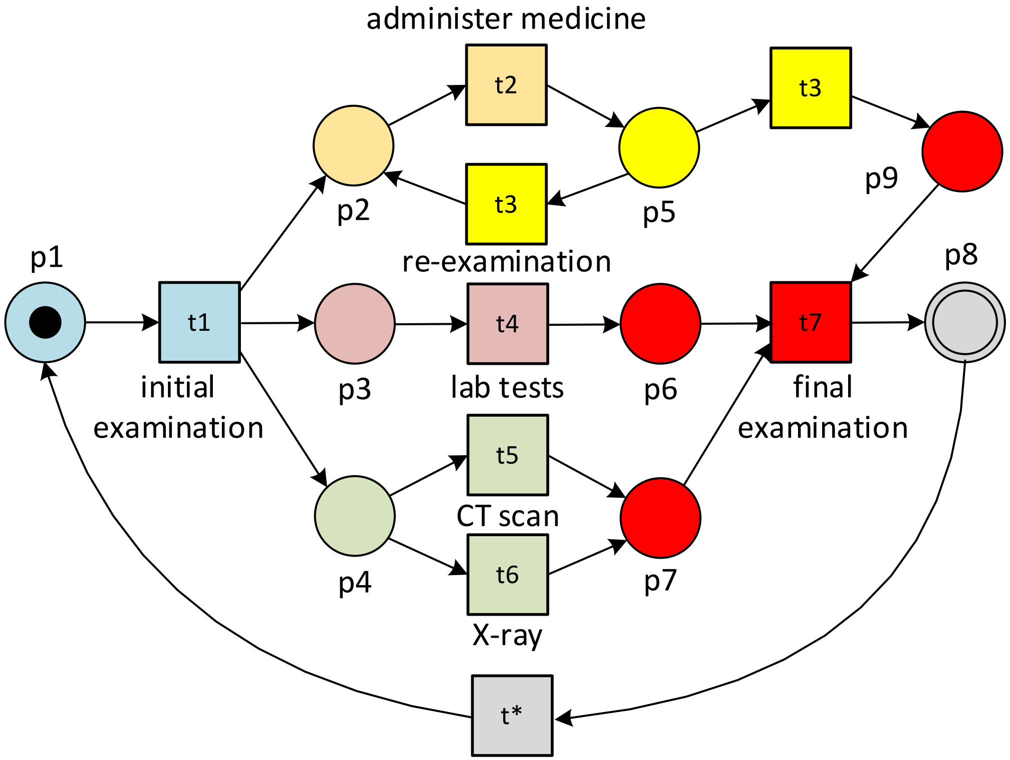
boundedness
safeness
deadlock free
liveness
well-formed

Clusters & Free-choice property



clusters
not free-choice

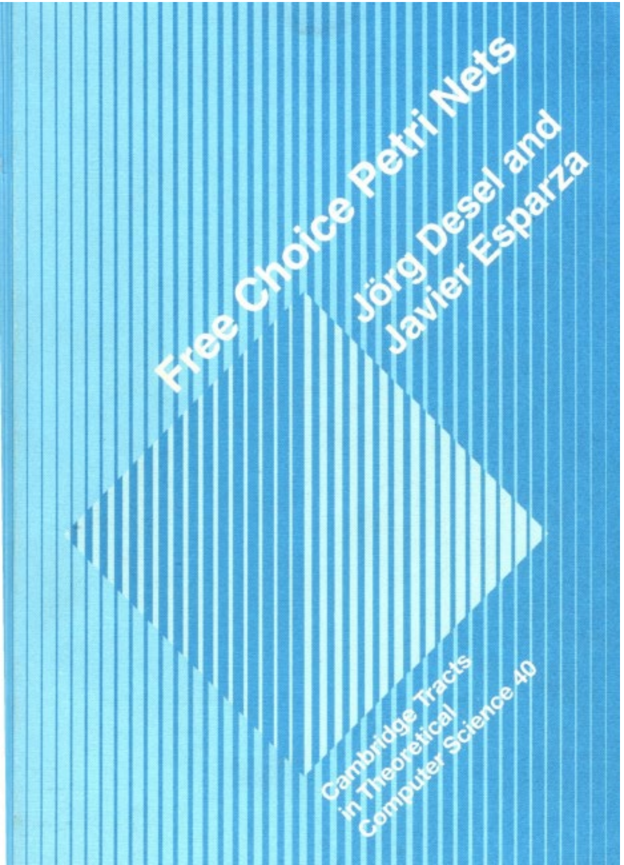
Clusters & Free-choice property



free-choice

We focus on free-choice and proper nets (no transitions without input or output places)

Strong & beautiful results for well-formed free-choice nets



- Commoner's theorem (siphons and traps)
- Coverability theorems (P/T-covers)
- Rank theorem (marking equation)
- Reduction rules (preserve well-formedness)

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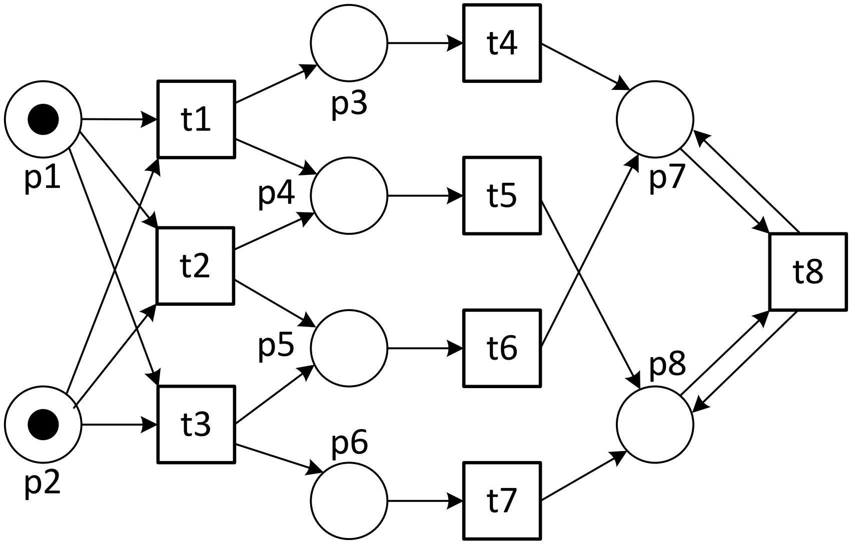
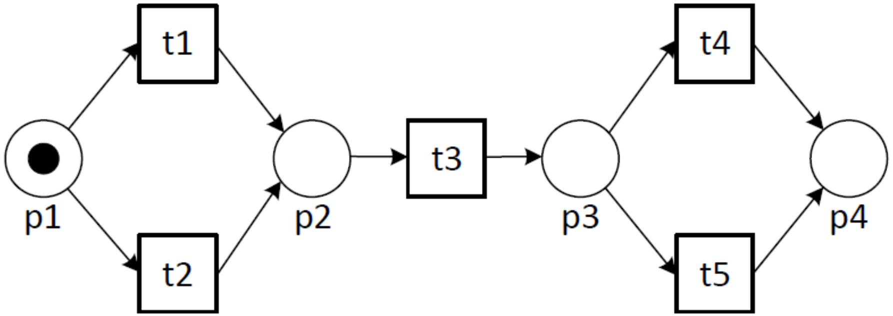
- Wil van der Aalst: Lucent Process Models and Translucent Event Logs. Fundam. Informaticae 169(1-2): 151-177 (2019)
- Wil van der Aalst: Reduction Using Induced Subnets To Systematically Prove Properties For Free-Choice Nets. CoRR abs/2106.03658 (2021)

Also deal with models like these

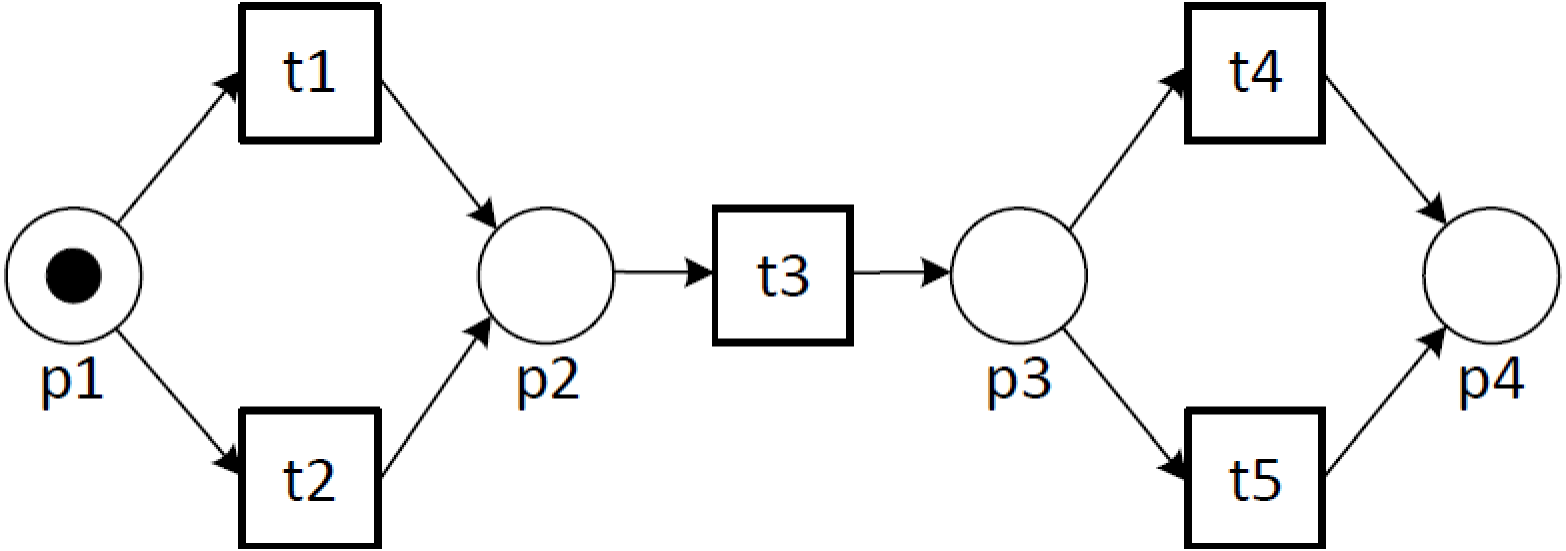
(non-well-formed or even not strongly connected)





Lucency

"A marked Petri net is lucent if there are no two different reachable markings enabling the same set of transitions, i.e., markings are fully characterized by the transitions they enable."



Lucent

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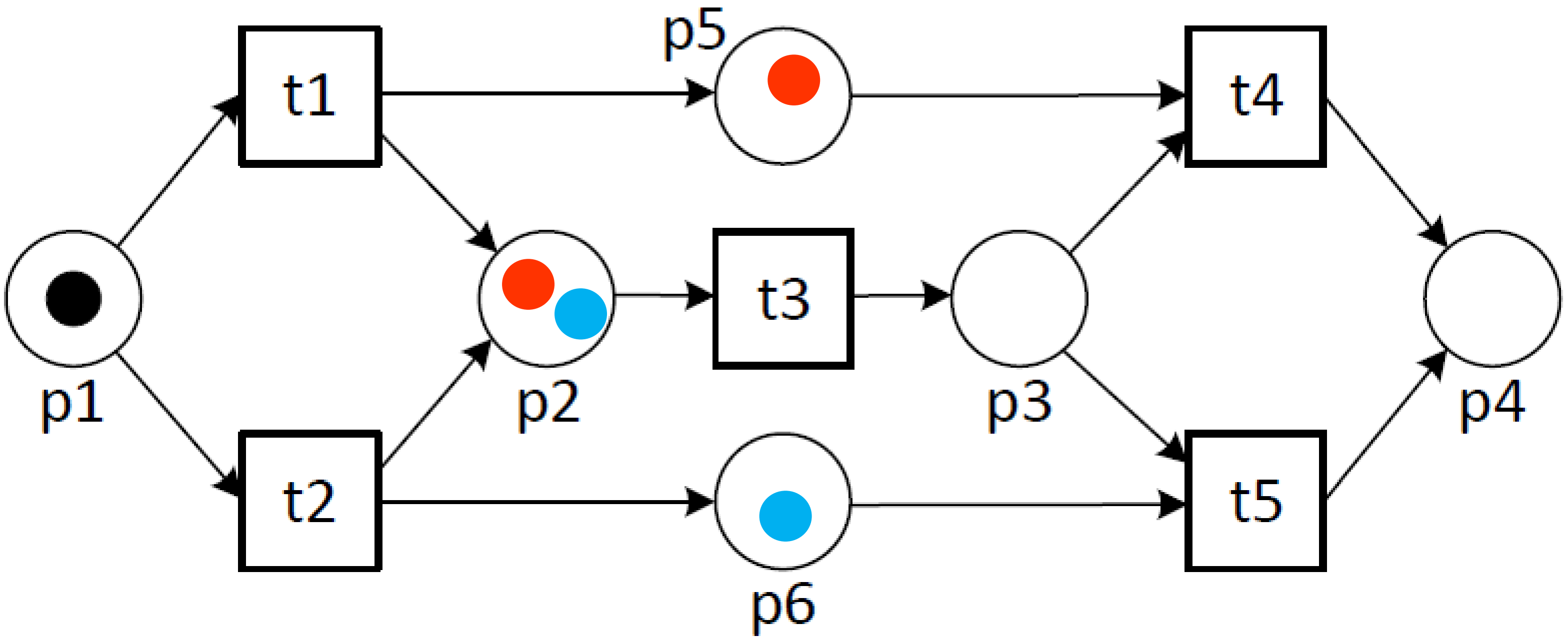

Non-lucent

Lucency

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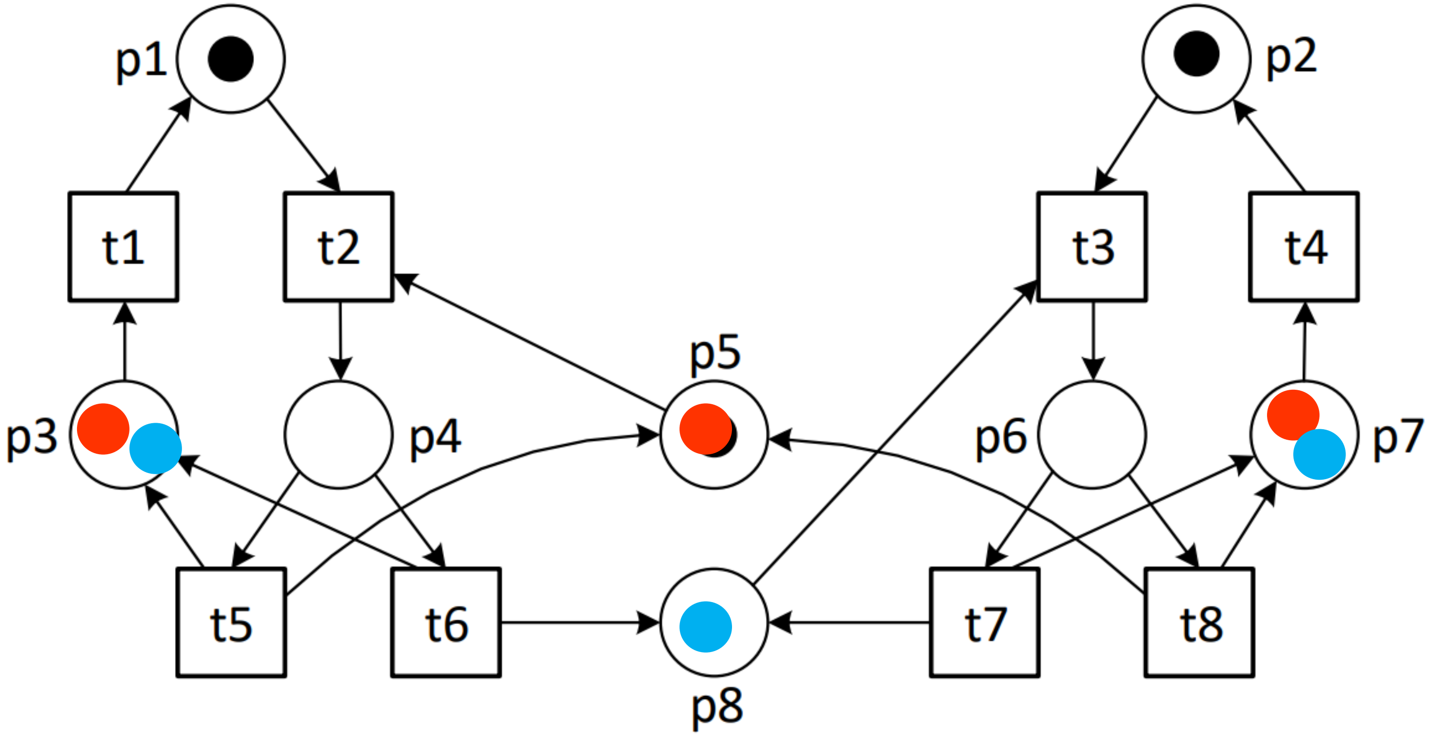

Non-lucent

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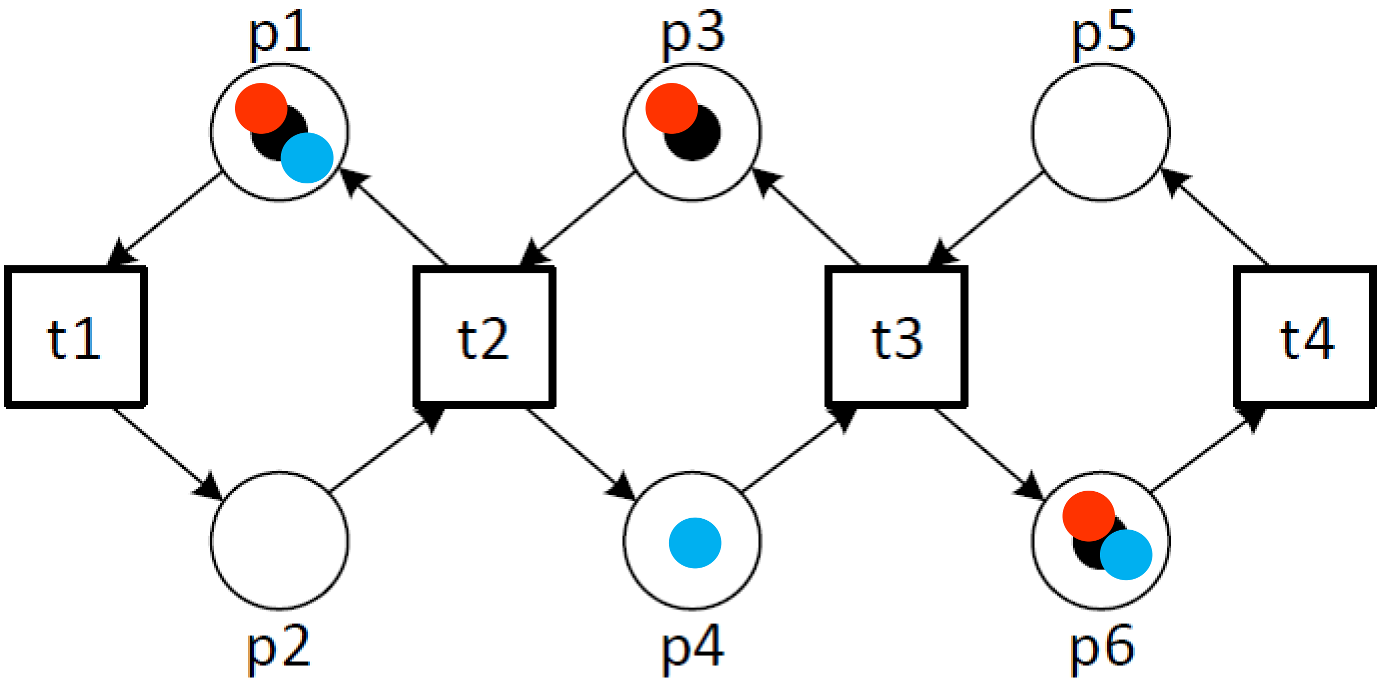


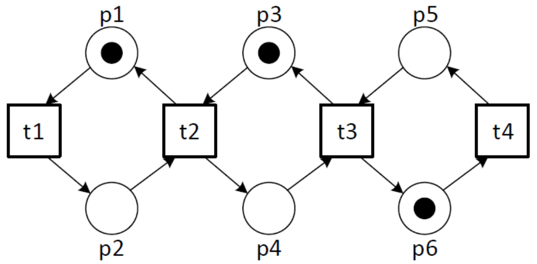
Non-lucent

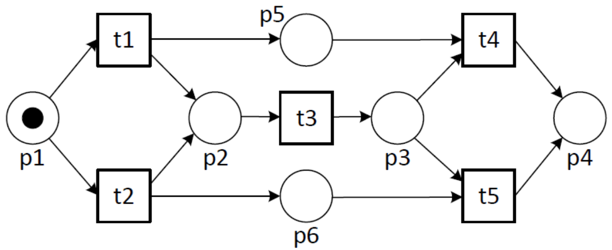
Lucency: Motivation

Also note translucent event logs.

Lucency

“A marked Petri net is lucent if there are no two different reachable markings enabling the same set of transitions, i.e., markings are fully characterized by the transitions they enable.”





- Seems to be related to free-choice property (but even marked graphs may be non-lucent).
- In [PN 2018] it was shown that any perpetual (live, bounded, home cluster) marked free-choice net is lucent.
- Here we consider free-choice nets that do not need to be well-formed.

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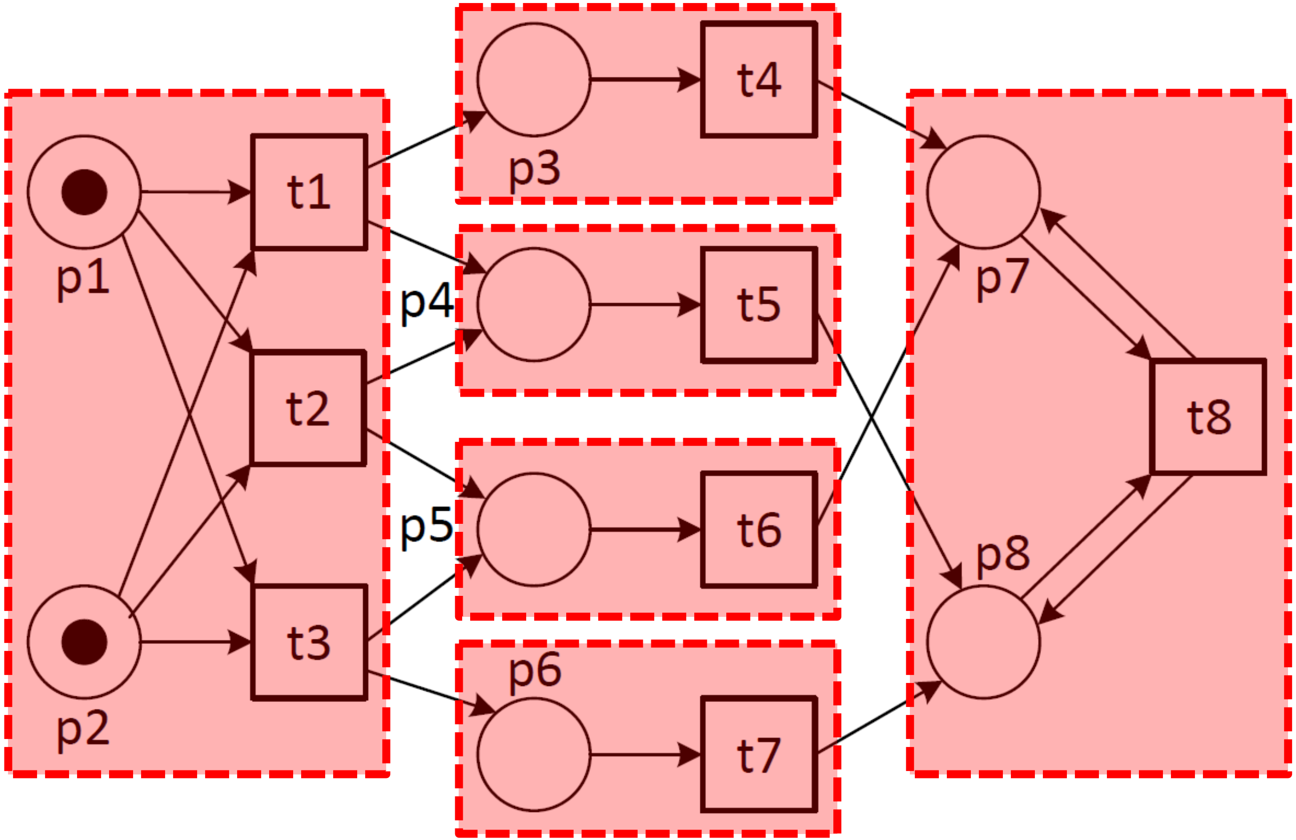
Home Clusters Ensure Lucency

- **Let (N,M) be a marked proper free-choice net having a home cluster. (N,M) is lucent.**
  - **A Petri net is proper if all transitions have input and output places.**
  - **A cluster is a minimal set of places and transitions such that for any transition in the cluster all input places are included and for any place all output transitions are included.**
  - **A cluster is a home cluster if from any reachable marking it is possible to reach a marking just marking the places in the cluster (once).**

Clusters



Home cluster

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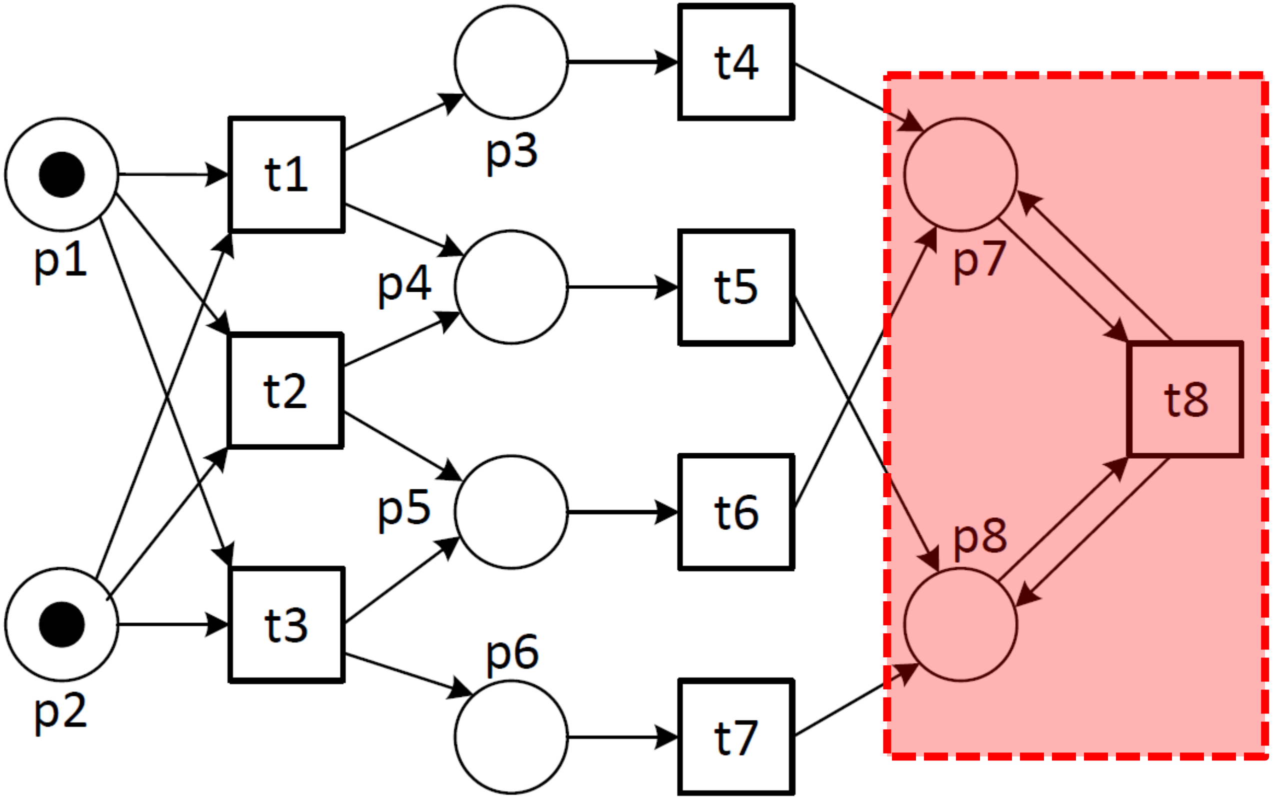

Hence, lucent!

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Remarkable result!

- Direct proof, i.e., <u>not</u> building on existing results.
- The nets do <u>not</u> need to be well-formed or strongly connected (i.e., existing results cannot be used)!
- Novel concepts such as (rooted) disentangled paths (paths where clusters appear once are safe) and conflict-pairs (witnesses of non-lucency).

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Some pointers

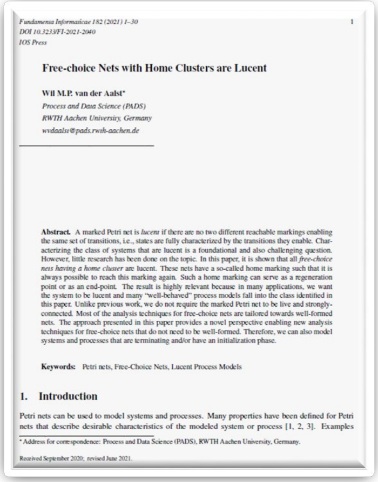
Definition 3.7. (Free-choice Net)
Let $N = (P, T, F)$ be a Petri net. N is *free-choice net* if for any $t_1, t_2 \in T$: $\bullet t_1 = \bullet t_2$ or $\bullet t_1 \cap \bullet t_2 = \emptyset$.

Definition 3.8. (Proper Petri Net)
A Petri net $N = (P, T, F)$ is *proper* if all transitions have input and output places, i.e., for all $t \in T$: $\bullet t \neq \emptyset$ and $t \bullet \neq \emptyset$.

Definition 4.1. (Lucent Petri nets)
Let (N, M) be a marked Petri net. (N, M) is *lucent* if and only if for any $M_1, M_2 \in R(N, M)$: $en(N, M_1) = en(N, M_2)$ implies $M_1 = M_2$.

Definition 5.1. (Home Clusters)
Let (N, M) be marked Petri net. C is a *home cluster* of (N, M) if and only if $C \in [N]_c$ (i.e., C is a cluster) and $Mrk(C)$ is a home marking of (N, M). If such a C exists, we say that (N, M) has a home cluster.

Corollary 6.7. (Complexity of Home Cluster Detection)
The following problem is solvable in polynomial time: Given a marked proper free-choice net, to decide whether there is a home cluster.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Some pointers

Theorem 5.5. (No Dominating Markings in Free-Choice Nets With a Home Cluster) Let (N, M) be a marked proper free-choice net having a home cluster C. For all $M' \in R(N, M)$: if $M' \geq Mrk(C)$, then $M' = Mrk(C)$.

Definition 5.8. ((Rooted) Disentangled Paths) Let $N = (P, T, F)$ be a Petri net. $\rho = \langle p_1, t_1, p_2, \ldots, t_{n-1}, p_n \rangle$ is a *disentangled path* of N if and only if ρ is a path of N ($\rho \in paths(N)$), $p_1 \in P$, $p_n \in P$, and for all $1 \leq i < j \leq n$: $[p_i]_c \neq [p_j]_c$ (i.e., ρ starts and ends with a place and does not contain elements that belong to the same cluster). A disentangled path is *Q-rooted* if $p_n \in Q$.

Lemma 5.11. (Rooted Disentangled Paths Are Safe) Let (N, M) be a marked proper free-choice net having a home cluster C. For any reachable marking, $M' \in R(N, M)$ and C-rooted disentangled path $\rho = \langle p_1, t_1, p_2, \ldots, t_{n-1}, p_n \rangle$: $M'(\{p_1, p_2, \ldots, p_n\}) \leq 1$.

Key idea: let tokens on path move towards home cluster.

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Some pointers

Definition 5.13. (Conflict-Pair)
Let (N, M) be a marked Petri net. (M_1, M_2) is called a *conflict-pair* for (N, M) if and only if

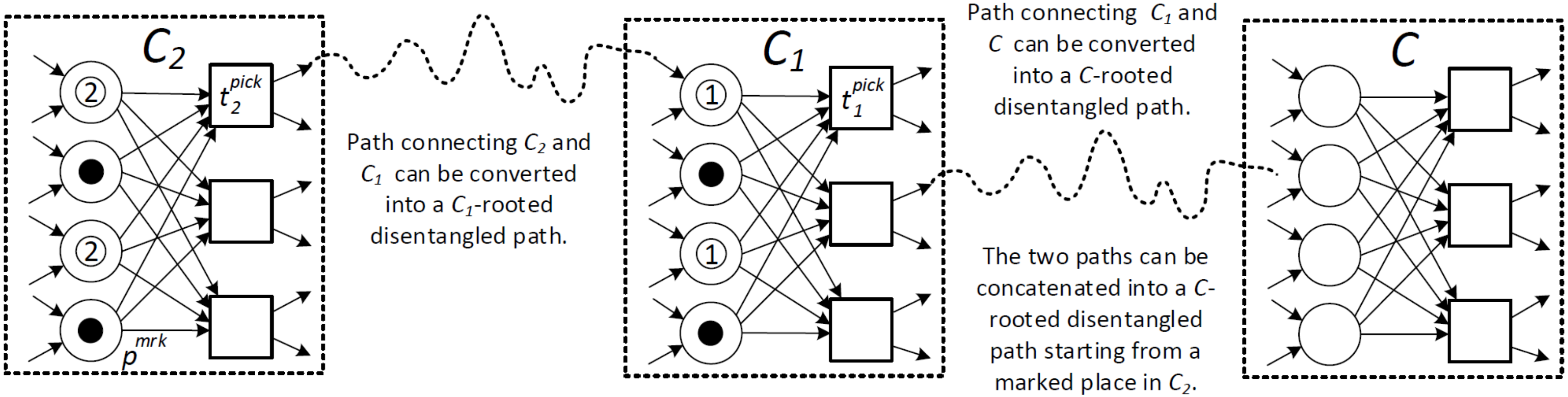
- M_1 and M_2 are reachable markings of (N, M) (i.e., $M_1, M_2 \in R(N, M)$),
- M_1 and M_2 are not dead (i.e., $en(N, M_1) \neq \emptyset$ and $en(N, M_2) \neq \emptyset$),
- $en(N, M_1) \cap en(N, M_2) = \emptyset$ (no transition is enabled in both markings),
- for all $t \in en(N, M_1)$: $M_2(\bullet t) \geq 1$, and
- for all $t \in en(N, M_2)$: $M_1(\bullet t) \geq 1$.

Lemma 5.14. (Nets Without Conflict-Pairs Are Lucent)
Let (N, M) be a marked proper free-choice net having a home cluster. If (N, M) has no conflict-pairs, then (N, M) is lucent.

Key idea: Never consume disagreement tokens and move towards home cluster.

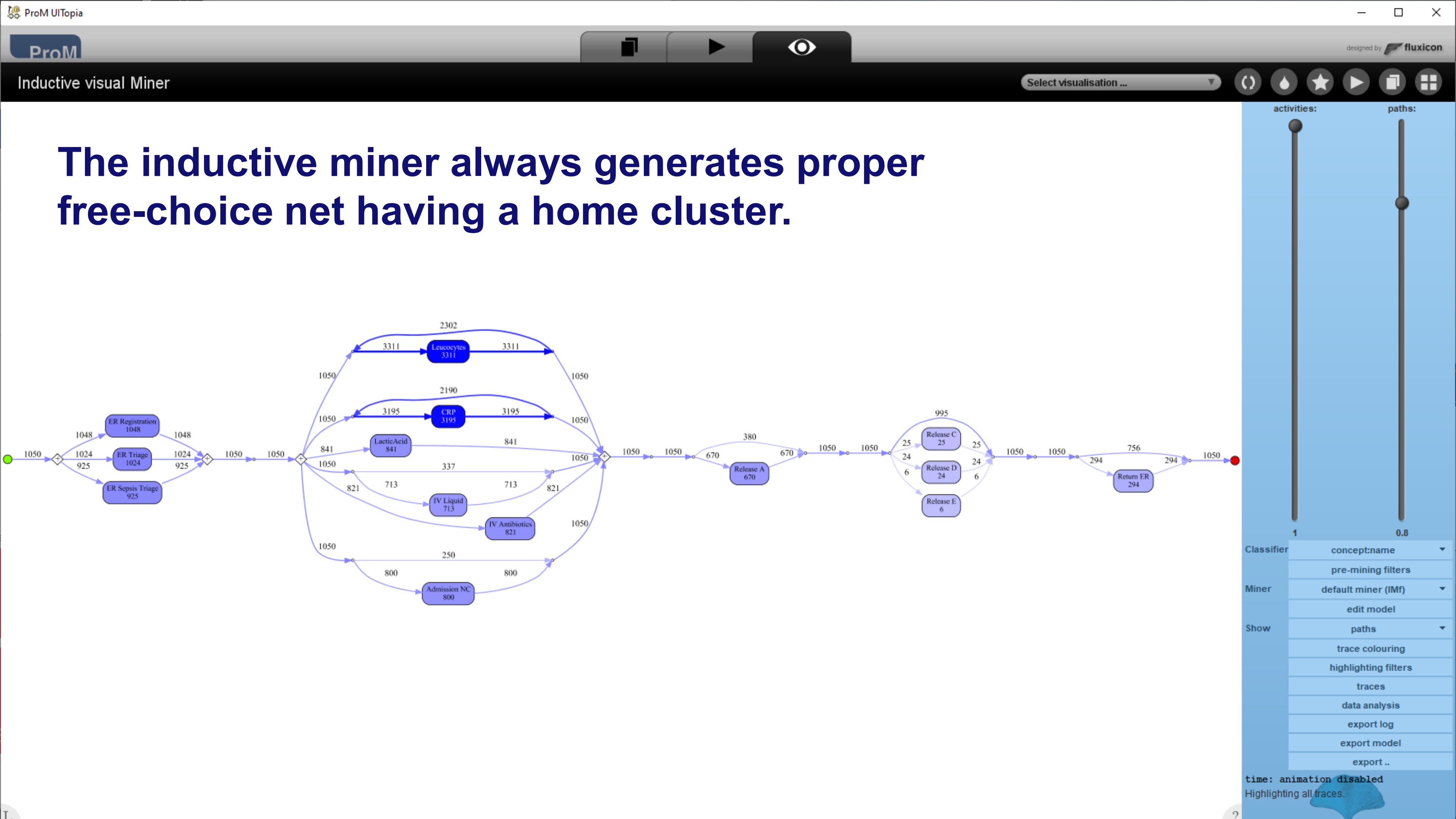
Some pointers

Theorem 5.15. (Home Clusters Ensure Absence of Conflict-Pairs)
Let (N, M) be a marked proper free-choice net having a home cluster. (N, M) has no conflict-pairs.



Corollary 5.16. (Home Clusters Ensure Lucency)
Let (N, M) be a marked proper free-choice net having a home cluster. (N, M) is lucent.

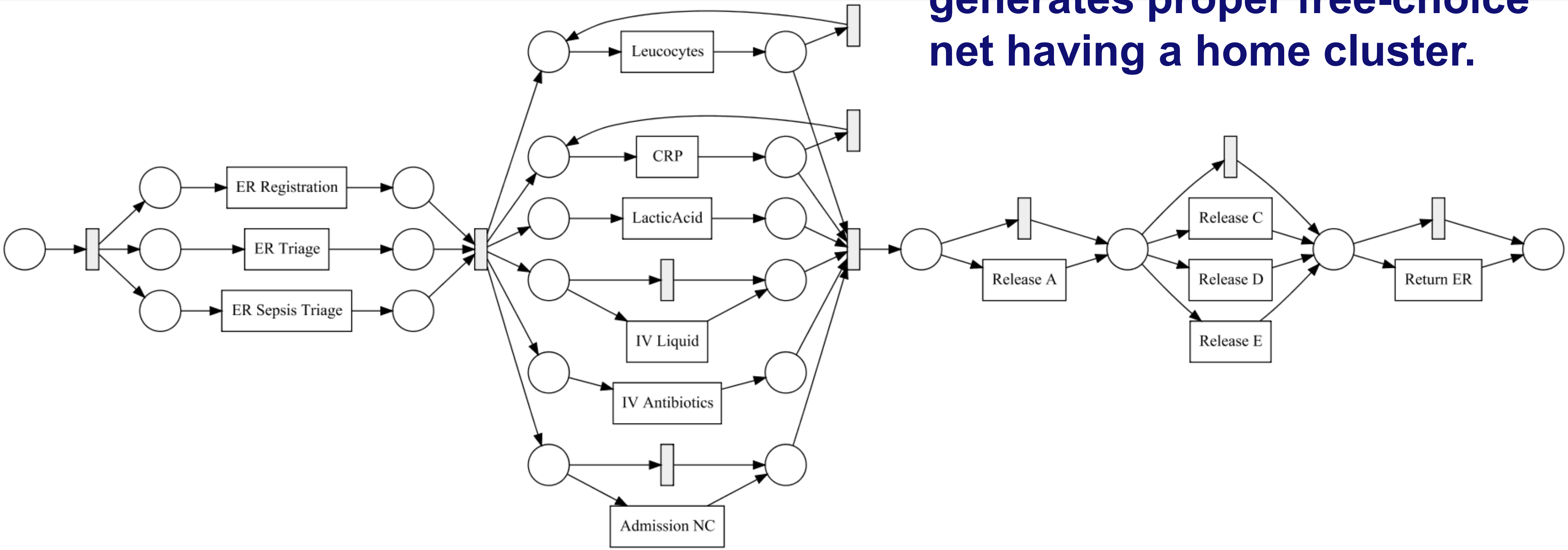
The inductive miner always generates proper free-choice net having a home cluster.



ProM UITopia

Petri net of Sepsis Cases - Event Log

The inductive miner always generates proper free-choice net having a home cluster.



Translucent Event Logs

The discovery of lucent process models becomes trivial

event	case	activity	time	enabled
e_1	1	a	09:22	$\{a\}$
e_2	1	b	09:34	$\{b, c\}$
e_3	2	a	09:45	$\{a\}$
e_4	2	c	10:12	$\{b, c\}$
e_5	1	c	10:17	$\{c\}$
e_6	1	e	11:06	$\{d, e\}$
e_7	2	b	11:22	$\{b\}$
e_8	3	a	11:55	$\{a\}$

event	case	activity	time	enabled
e_9	3	c	12:13	$\{b, c\}$
e_{10}	2	d	12:18	$\{d, e\}$
e_{11}	2	b	13:32	$\{b, c\}$
e_{12}	2	c	13:43	$\{c\}$
e_{13}	3	b	13:52	$\{b\}$
e_{14}	2	e	14:17	$\{d, e\}$
e_{15}	3	e	14:20	$\{d, e\}$
...	...	...	...	...

enabling set determines state

W. van der Aalst. Lucent Process Models and Translucent Event Logs. Fundamenta Informaticae, 169(1-2):151-177, 2019.

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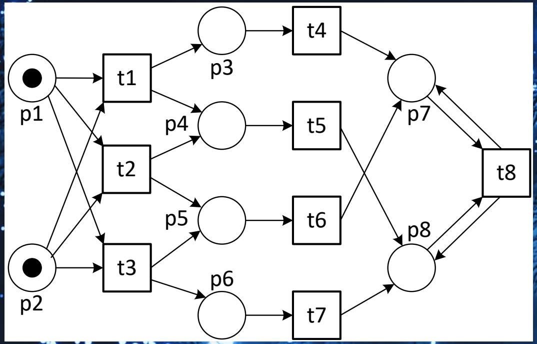
Conclusion

Free-Choice Nets with Home Clusters are <u>Surprisingly</u> Lucent !!



New concepts, not well-formed, direct proofs!

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